

Impacts of Loop Flow on Electricity Market Design

Chin Y Choo, *Student Member, IEEE*, Nirmal-Kumar C Nair, *Member, IEEE*, Bhujanga Chakrabarti, *Member, IEEE*

Abstract—The process towards conforming with standardized market design (SMD) in electricity markets like New Zealand, PJM, ISO-NE and ERCOT, includes features of locational marginal pricing and financial transmission rights. This paper presents occurrence of price volatilities in LMP-based market typically during transmission congestions in a loop network which is termed as spring washer effect in New Zealand Electricity Market (NZEM). The loop flow or parallel path flow that causes this impact is analyzed using DC load flow model. Financial transmission rights (FTR), a financial hedging instrument, is recommended for electricity markets by SMD. The paper also demonstrates the potential impacts of loop flow control devices on FTR.

Index Terms— Deregulation, electricity markets, financial transmission rights (FTR), loop flow, linear programming, locational marginal pricing (LMP), parallel path flow, standardized market design (SMD), thyristor controlled series capacitors (TCSC).

I. INTRODUCTION

Restructuring of the electrical power industry, motivated by the need to induce competition and efficiency, has led to the development of several market mechanism and pricing models. As electricity markets worldwide mature, various rule changes are constantly being made to address the technical and market challenges. All these new market mechanisms and technical procedures have to be consistent during normal trading periods and ensure maintain system integrity.

A successful market design challenge is arriving at intersection between seemingly independent issues like commercial incentives, reliability rules and network interactions [1]. The challenge is therefore to ensure market fairness and efficiency for short-term and generating clear signals for long-term lumpy transmission upgrade investment decisions. Electricity markets, worldwide, are currently evolving towards implementing some form of the standardized market design (SMD) aimed at providing greater market flexibility and effectiveness in achieving congestion relief [2]. One of the recommendations of SMD is implementing locational marginal pricing (LMP) for energy markets similar to the already functional markets of New Zealand Electricity Market (NZEM), PJM & NYISO. In comparison to other pricing mechanisms like Uniform Marginal Pricing (UMP) and Zonal Marginal Pricing (ZMP), LMP-based approach has demonstrated to be effective in providing transparent economic signals particularly in market-based congestion

management [2],[3]. But, LMP price during certain market periods in NZEM were observed to be highly volatile and has received considerable attention from market participants [4]. This pricing characteristic, which is closely related with congestions occurring in loops, is described by the NZEM participants as “Spring Washer” effect [5],[6]. These large price variations affect adjacent nodes within a constrained loop network. From the times of vertically integrated operation for interconnected systems, loop flows or parallel path flows which can be classified both as major and minor loop flows, have been extensively analyzed and documented [7]. In deregulated market operation, LMP distributions typical of spring washer have been attempted to study network examples that have simple loop structures and also complex situations arising due to congestions in adjacent lines [3]. The market mechanism to handle the short term LMP price extremes for hedging the risk of congestion charge is by deploying Financial Transmission Rights (FTR) in LMP-based markets [8]. FTR has been operational in PJM and ISO-NE markets and is being planned to be instituted in other LMP-based energy markets. It is well known that loop flow distribution can potentially be altered using devices like phase shifters, Thyristor Controlled Series Capacitors (TCSC) and other Flexible AC Transmission Systems (FACTS) [9-11]. Thus naturally the presence of such devices should change loop flow patterns thereby affecting congestion, LMP nodal prices and FTR receipts.

The thrust of this paper is to demonstrate the LMP volatility due to congestions and its direct impact on FTRs in a loop network. Furthermore, we will highlight the possible impact of TCSC operation that can create different loop flow patterns thereby affecting FTR rentals as well. In markets that are moving towards LMP, from other structures like UMP and ZMP, these effects could potentially be experienced in future trading periods. In this paper we look only at the LMP-based energy market and FTR market for transmission congestion hedging. Ancillary Services (AS), which is another important aspect of SMD, is not considered in this research. In this paper we have not proposed any enhancement to existing pricing models aimed at accommodating loop flow impacts even though it would be the ideal goal. However, a recent publication proposes transmission rights framework based on both capacity and line admittance [12]. This paper is organized as follows. Section II explains loop flow using a DC load flow model. The occurrence of spring washer due to loop flow and its impact on LMP will be demonstrated in Section III and

Section IV presents the impact of spring washer on the final prices in the NZEM. The impact of loop flow control devices on FTR will be further illustrated in Section V. Conclusions, appendix and references are arranged in subsequent sections.

II. LOOP FLOW

A closed-loop transmission network that interconnects with other systems will experience a phenomenon called loop flow. Loop flow occurs when some portions of a scheduled power are distributed into other branches that are adjacently connected. This happens as power flow is strictly governed by Kirchoff's Law. Loop flow is mathematically defined as the difference between the scheduled transaction and actual loading of the line connecting bus i and j . It can also be referred to as the parallel path flow, unscheduled flow or circulating flow. The loop flow behaviour of a system mainly depends on the network topology. Based on the network structure, there are two possible kinds of loop flows known as major and minor loop flows. Major loop flow is the difference between the actual and the scheduled power flow which is measured on the path that cuts the major loop or the main grid of the system. On the other hand, minor loop flows are localized loop flows that exist between smaller loops that are closely connected. In real practical situations, loop flow with the order of more than half of the scheduled flows in a network has been reported. Some lines in WSCC (Western System Coordinating Council) network have experienced loop flow of up to 75% of their scheduled capacity [7].

DC load flow model is a robust, practical approximation for fully coupled load flow expressions and is used throughout all discussions in this paper. This is because we are focusing only on the energy market. This DC model is derived based on the following assumptions:

- Effect of reactive power is not considered.
- Since line resistance R_{ij} being usually smaller compared to line reactance X_{ij} , it is therefore neglected in the network. In short, transmission loss is not considered in the model.
- Phase angle difference across line $(\theta_i - \theta_j)$ being generally small, can be further simplified so that $\sin(\theta_i - \theta_j) \approx (\theta_i - \theta_j)$.
- By neglecting reactive flows in the system, it is reasonable to assume $V_i \approx V_j = 1 \text{ pu}$.

Using the above assumptions for a network, we arrive at a linear algebraic equation as shown in equation (1) for a line. Unlike the fully coupled AC model having non-linear algebraic equations that requires iterative solution procedures, the DC model solution is direct and supports the principle of superposition. This aids in arriving at quicker solution for electricity pricing optimization models, which is essential for the functioning of a real-time electricity market.

$$P_{ij} = \frac{\theta_i - \theta_j}{X_{ij}} \quad (1)$$

Fig. 1 clearly illustrates a loop flow in a 3-bus network using two generators and a load of 3000 MW. Suppose generator 1 is required to schedule a total power of 2000 MW to the load at bus 3. But in actual practice, only 70% of the scheduled power flows from bus 1 to bus 3. The remaining 30% of power will flow along the non-contract paths of lines 1-2 and 2-3. This remaining flow is known as loop flow. In fact, the behavior of electricity in a system is basically governed by Kirchoff's Law and not through contractual terms. This means that power flowing in a loop must always respect the Kirchoff's loop constraint as shown by equation (2).

$$\sum_{j>i} P_{ij} X_{ij} = \theta_i - \theta_j = 0 \quad (2)$$

The amount of loop flow along a line depends mainly on the physical parameters of the network which is basically the line impedances. The line impedance between two buses is usually determined by its length, number of parallel paths, voltage level and number of series elements used such as transformer.

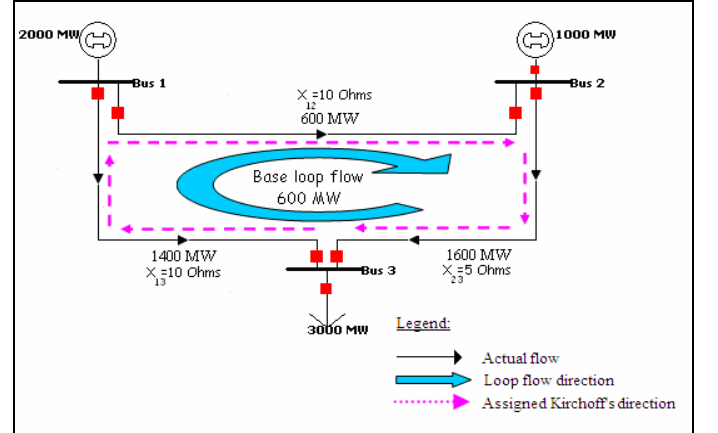


Fig. 1. Demonstration of Loop flow

To control major loop flows many power systems utilize power flow controllers like phase shifters, TCSC and DC lines. All these devices in the bulk transmission system can be controlled correspondingly to change the power flow patterns. Although these devices help in redistribution of loop flows, it does not overcome loop flows completely.

III. LOOP FLOW IMPACT ON LOCATIONAL MARGINAL PRICING

LMP is defined as a nodal price of supplying the next unit MW of power at a specific node based on generation marginal cost and physical aspects of the transmission system [8]. An optimal electricity price is usually obtained when the total price matches the electricity supply with the demand. In the NZEM, LMP pricing model incorporates co-optimization of energy with reserves and also considers transmission losses [13]. Security margin can also be included in such pricing

models [14]. For simulations in this paper, we have only considered energy in our pricing model to compute the nodal prices and the pricing details can be found in Appendix I. The optimization problem is solved using a CPLEX solver in GAMS (Generalised Algebraic Modelling System) [15].

LMP is also sometimes known as shadow price, dual price or marginal cost which is usually measured in \$/MWh. The price for a demand at a specific node is governed by various shadow prices associated with one or more linear constraints. With LMP, energy retailers have the option to buy electricity at their bidding prices. This has significant influences on the required amount of generation for optimum dispatch because this determines new power flow pattern which reflects certain amount of loop flows in transmission lines. In some cases, the increased flows exceed the thermal capacity limit causing significant impacts on LMP market. These volatile price excursions create undesirable outcomes for some market participants in the short run.

A. Spring washer effect

Locational marginal pricing provides market participants a clear electricity price indicator for all nodes on the grid. Generally, price differentials between source and sink nodes for a congested radial systems are well defined. However, the equitable determination of prices for nodes becomes more complicated for a loop network due to the presence of loop flows. Based on a market participant's complaint due to the unusually high LMP price during a trading period of 24th April 2004 in NZEM operation, the Electricity Commission of New Zealand instituted a working group to closely look into it [16]. Transmission constraint along a loop was the main cause for this price volatility which is known as spring washer. The term spring washer effect reflects the inconsistent pattern of electricity prices that result within a loop network. Price spikes affect nodes that are situated along a constrained transmission line and followed by decreasing prices when heading towards the original node where it started. Some analytical descriptions for spring washer on simple cyclic loop have been attempted [5],[6] but no general analysis that fits all possibilities of loop formations has been known to be reported.

We will demonstrate this effect on a sample test system. The corresponding transmission line reactance and thermal limit are shown in Table I. The data has been artificially set-up to demonstrate the electricity pricing issues that have occurred in NZEM. To explain the spring washer we will first explore the behavior of LMP prices in an unconstrained cyclic transmission loop. A derivation of prices around the four-bus cyclic transmission loop with two generators and loads assuming a simple DC load flow model is illustrated in Fig. 2.

As shown in Fig. 2, generator at node 1 is supposed to schedule 200 MW to node 2 and 450 MW to node 3. But due to Kirchoff's Law, power flow does not follow according to the contractual terms but travels with respect to its line reactance with actual flows as shown in the figure. This results a base loop flow of 138.71 MW in a clockwise direction. To respect the Kirchoff's Law, loop equation with the assigned clockwise direction as stated previously in equation (2) must always be satisfied.

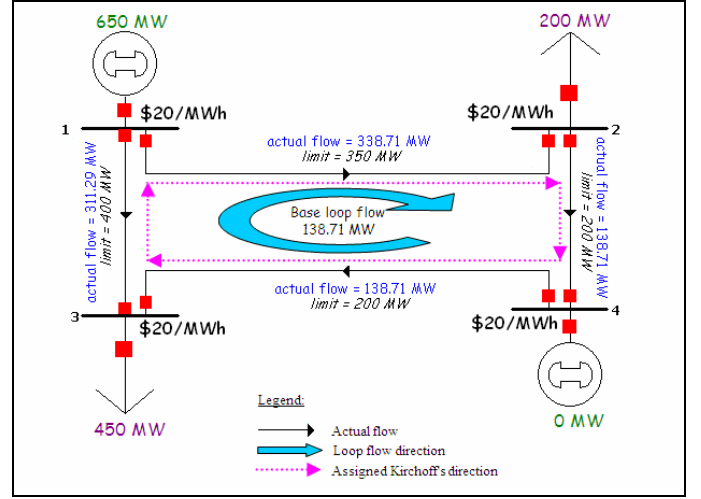


Fig. 2. Unconstrained system dispatch

TABLE I
TRANSMISSION NETWORK DATA

From	To	Reactance (Ω)	Line Limit (MW)
1	2	0.1	350
2	4	1	200
4	3	1	200
1	3	1	400

The details for generation offer, price and demand data are given in Table II. To meet the total demand power of 650 MW, the optimal solution is to have generator at node 1, being the cheapest generator, to supply all required power to these two nodes. Based on this dispatch, the system is subjected to operate under a non-stress condition. Mathematically, nodal price at bus j , for this example, is driven by different shadow prices if any of its constraint is binding as shown by equation (3), which can be derived from the pricing model given in Appendix I.

$$\lambda_j^P = \lambda_i^P + \phi_{ij} - \tau_{ij} \quad (3)$$

where

λ_j^P = nodal price at bus j

λ_i^P = nodal price at bus i

ϕ_{ij} = shadow price due to line capacity constraint

τ_{ij} = shadow price due to power flow using DC-based model.

Since none of the constraints in equation (3) is binding all nodes in the loop will have equal marginal price of \$20/MWh. If losses had been considered in our model whereby transmission resistances are taken into account, the LMP prices at all nodes would vary slightly instead. For our example, both generation and demand sides would have the same total prices of \$13000 as given in Table III and IV.

TABLE II
GENERATION OFFER, PRICE AND DEMAND DATA

	Node 1	Node 2	Node 3	Node 4
Maximum Generation offer (MW)	800	-	-	200
Price offer (\$/MWh)	20	-	-	100
Load (MW)	-	200	-	450

TABLE III
OPTIMAL DISPATCH AND GENERATION PRICE FOR UNCONSTRAINED CASE

Node	Generation dispatch (MW)	Generation Price (\$/MWh)	Total Generation Price (\$)
1	650	20	13000
4	0	100	0
$\Sigma=$			13000

TABLE IV
OPTIMAL DEMAND PRICE FOR UNCONSTRAINED CASE

Node	Demand (MW)	Demand Price (\$/MWh)	Total Demand Price (\$)
2	200	20	4000
3	450	20	9000
$\Sigma=$			13000

Let us now consider the same network previously but with an increased demand at node 3 by an amount of 50 MW resulting in a total demand of 700 MW. To meet the new demand, the system has a new generation dispatch with actual flow patterns as shown in Fig. 3. As a result of the increased loop flow of 150 MW, line 1-2 reaches its thermal limit of 350 MW and does not allow anymore injection of power from generator at node 1. Therefore, generator at node 4 is committed to supply an additional amount of 7.5 MW to satisfy the demand requirement. Please note that the current dispatch consists of two marginal generators. Due to the binding constraints, nodal prices for nodes 2 and 3 are set respectively at \$140/MWh and \$60/MWh. Please refer to Appendix I for more details of the pricing equations which are formulated correspondingly in GAMS.

It is obvious that the high spike that is experienced by node 2 is due to the congestion of line 1-2. The impact of transmission constraint on the loop prices will also be reflected throughout the network causing prices to behave differently. This scenario is often described by the NZEM participants as 'spring washer effect'.

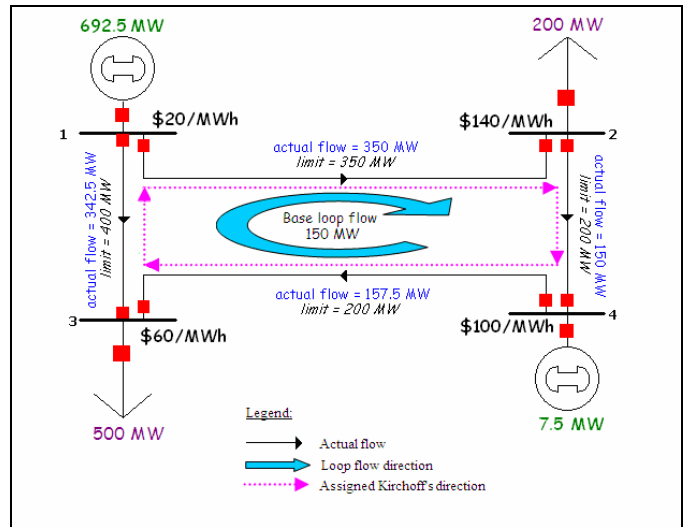


Fig. 3. Constrained system dispatch

As shown in Table V and VI, the total generation price for supplying power to loads increases to \$14600 whereas total demand price to be paid by loads increases to \$58000 resulting a price difference of \$43400 which is accounted for the total congestion rentals.

TABLE V
OPTIMAL DISPATCH AND GENERATION PRICE FOR CONSTRAINED CASE

Node	Generation dispatch (MW)	Generation Price (\$/MWh)	Total Generation Price (\$)
1	692.5	20	13850
4	7.5	100	750
$\Sigma=$			14600

TABLE VI
OPTIMAL DEMAND PRICE FOR CONSTRAINED CASE

Node	Demand (MW)	Demand Price (\$/MWh)	Total Demand Price (\$)
2	200	140	28000
3	500	60	30000
$\Sigma=$			58000

From electricity market viewpoint, customer at node 2 seems to be disadvantaged in having to pay more compared to customer at node 3 simply due to the increased demand at node 3 by 30 MW. From fairness perspective, node 2 should not bear this steep price hike since node 2 is located much closer to generator at node 1 compared to node 3, by looking at its line reactance of 0.1Ω as shown in Table 1.

It is clearly shown that loop flow can instigate a highly volatile LMP pricing which closely relates to the occurrence of spring washer effect. It also implies that prices can vary

significantly whenever one or more lines in the loop network are constrained. All load entities would prefer to effectively hedge these congestion charges due to loop flow while participating in the wholesale electricity market.

IV. SPRING WASHER EFFECT IN NZEM

Undesirable Trading Situations (UTS) in NZEM were allegedly reported by market participants in relation to high prices at certain trading periods in 2004. The Constraint Issues Group (CIG) was formed to investigate these extreme prices [17]. We have analyzed several trading scenarios, which show typical characteristics of spring washer effect, from the dataset provided by Electricity Commission, New Zealand [18]. Here, we present the findings for two scenarios, 19th of March and 21st of August 2004, for North Island 220kV cyclic loop. Some of them have been reported as UTS [17].

Price volatility was observed for trading period 17 on 19th of March 2004 which happens to be caused by the line constraint connecting between Whakamaru (WKM) and Tokaanu (TKU). There is an increasing price pattern from TKU to WKM in a clockwise direction causing a large price difference between these two nodes thus reflecting the spring washer characteristics, as shown in Fig. 4.

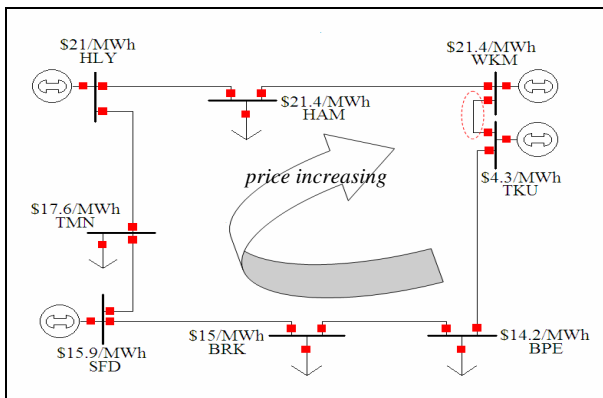


Fig. 4. Final prices for trading period 17 on 19th March 2004 due to the constrained line connecting between Whakamaru and Tokaanu

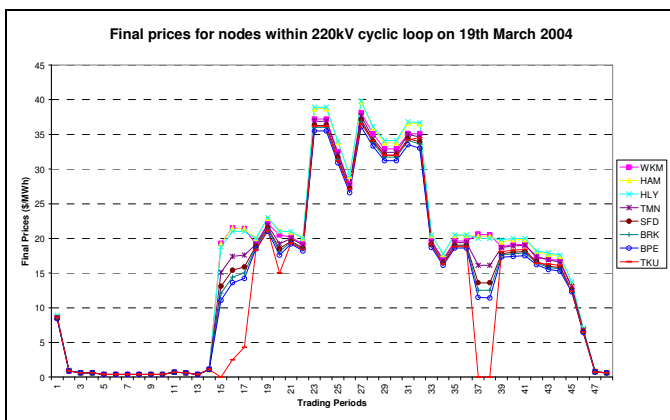


Fig. 5. Final prices for nodes within 220kV North Island transmission cyclic loop for all trading periods of 19th March 2004

On the same day during the other trading periods 15, 16, 17, 20, 37 and 38, the prices difference between the nodes followed spring washer pattern summarized in Fig. 5.

UTS for same loop was reported for the trading period 22 on 21st August 2004. This spring washer like effect is caused by the line constraint of Tokaanu (TKU) to Bunnythorpe (BPE) link. This scenario exhibits a different price pattern where the price began to increase from TKU to BPE in an anti-clockwise direction as shown in Fig. 6. The price difference between TKU and BPE for trading period 22 is much more significant than the volatility observed on March 19th.

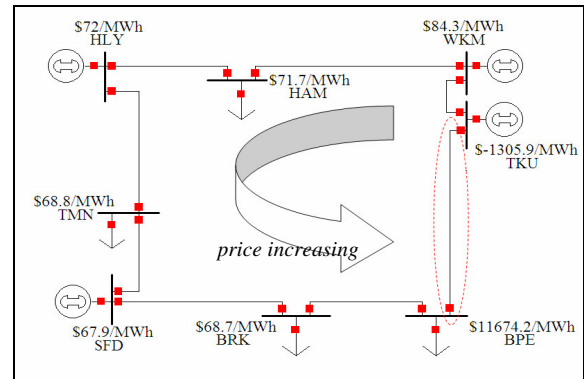


Fig. 6. Final prices for trading period 22 on 21st August 2004 due to the constrained line connecting between Tokaanu and Bunnythorpe

The price volatilities and spread between TKU and BPE observed in trading periods 22 and 23, as shown by Fig. 7, can be attributed to spring washer effect.

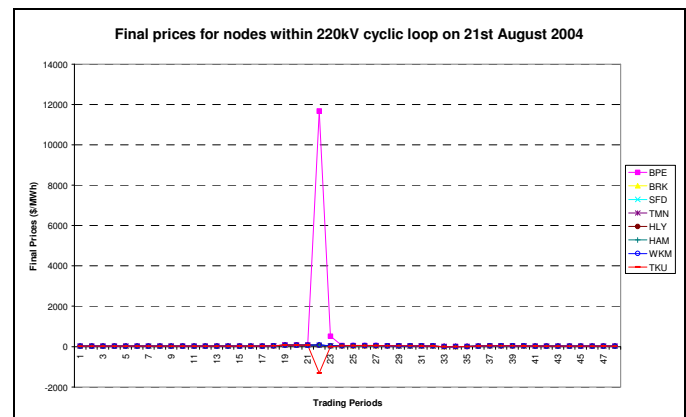


Fig. 7. Final prices for nodes within 220kV North Island transmission cyclic loop for all trading periods of 21st August 2004

V. IMPACT OF LOOP FLOW ON FINANCIAL TRANSMISSION RIGHTS IN LMP-BASED MARKET

To dampen the price volatility in LMP-based market, financial transmission rights (FTR) are proposed to ensure efficient allocation of transmission access rights. It is also one of the key elements proposed in SMD. By definition, FTR is a financial mechanism that is generally used to hedge congestion risk by introducing a disbursement of congestion rents in a point-to-point contract [2],[8]. FTR scheme allows a market participant to purchase a right to transfer power over a transmission path and a right to receive credit to counteract the congestion charge [8].

Transmission right allocation models have also evolved through several stages. The most common FTR used is called Point-To-Point FTR (PTP FTR). PTP FTR can be an

obligation type in which the FTR holder is responsible for the negative congestion rent. On the other hand, the holder for PTP FTR option is never liable for any negative congestion.

In the following sub-section, the effect of loop flow on LMP is described further with FTR application. The use of power flow controllers such as TCSC in the network are clearly illustrated to show the changing effect of loop flow which causes LMP differences to vary thus resulting different FTR values.

A. Effects of Loop Flow Controllers

With the ongoing expansion and growth of electric power industry, the use of power flow controllers is becoming more prevalent in modern power systems. The needs for system enhancement trigger the advancement of Flexible AC Transmission System (FACTS) technology which offers superior features in increasing system stability and performance. In practice, one can install a TCSC especially for a long transmission line to reduce the total effective line impedances to obtain the benefit of maximum power flow transfer. Its impact on loop flow can be estimated [10]. Clearly, the use of these devices can introduce significant implications on LMP and hence FTR allocations.

The following examples portray the effects of TCSC operation in LMP-based market with FTR. TCSC will be incorporated on line 1-3 with a specified controllability range of -0.5 to +0.5 times the nominal line reactance. To entitle specific charges to FTR holders, congestion charge is computed as in equation (4).

$$C = P \times (\lambda_{sink} - \lambda_{source}) \quad (4)$$

where C is the total congestion charge (\$)

P is the amount of FTR in a specified path (MW)

λ_{sink} is the nodal price for sink node (\$/MWh)

λ_{source} is the nodal price for source node (\$/MWh)

Firstly, consider a case without TCSC for the same network of the previous section. Table VII presents the corresponding FTRs for the constrained case discussed in the previous section and shown in Fig. 3.

Since line 1-2 is constrained, LMP prices within the loop vary thus creating different congestion charges. The incurred total congestion charge for system is \$55700 for that particular dispatch. Please note that since it is PTP FTR option, negative credit is not considered. To hedge the risk of congestion for high price spike of \$42000 due to congestion, market participant at node 2 will probably be interested to purchase the access rights on line 1-2 for long term hedging benefits. Market participants at node 3 might also be interested in hedging against its own possible price volatilities.

In the second case as shown in Fig. 8, a TCSC is installed on line 1-3 which is controlled such that the line reactance is reduced to 0.9Ω. This in turn changes the power flow patterns and causes system to operate without any congestion. Loop flow is also reduced to 143.44 MW compared to previous case as in Fig. 3. As a consequence, LMP prices for all nodes are identical thus no congestion charge is imposed on any of the nodes as shown in Table VIII.

TABLE VII
CONGESTION COSTS FOR A CONSTRAINED CASE (WITHOUT TCSC)

Sink Node	Source Node	λ_{sink} (\$/MWh)	λ_{source} (\$/MWh)	P (MW)	C (\$)
2	1	140	20	350	42000
4	2	100	140	150	-6000
3	4	60	100	157.5	-6300
3	1	60	20	342.5	13700
Total congestion charge					55700

TABLE VIII
CONGESTION COSTS (WITH TCSC, $X_{1,3}=0.9\Omega$)

Sink Node	Source Node	λ_{sink} (\$/MWh)	λ_{source} (\$/MWh)	P (MW)	C (\$)
2	1	20	20	343.44	0
4	2	20	20	143.44	0
3	4	20	20	143.44	0
3	1	20	20	356.56	0
Total congestion charge					0

Consider the case as shown in Fig. 9 where the TCSC has significantly reduced the reactance of line 1-3 to 0.5Ω. This results a different flow pattern, thus producing new LMP prices and congestion charges as shown in Table IX. By comparing the first case as in Table VII, the total congestion charge is now reduced to \$49637.40. At this moment, line 1-3 is experiencing the highest price which is, as expected, to be consistent with its larger demand. It also causes LMP at node 2 to be higher than \$20 but it is definitely not as high when compared to a typical spring washer scenario.

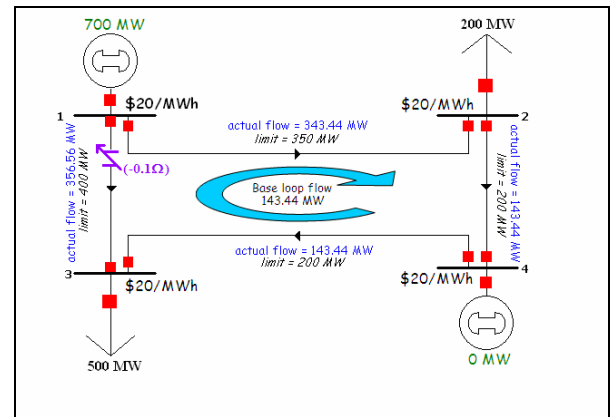


Fig. 8. System with TCSC in line 1-3 ($X_{1,3}=0.9\Omega$)

TABLE IX
CONGESTION COSTS (WITH TCSC, $X_{1,3}=0.5\Omega$)

Sink Node	Source Node	λ_{sink} (\$/MWh)	λ_{source} (\$/MWh)	P (MW)	C (\$)
2	1	27.27	20	272.73	1982.75
4	2	100	27.27	72.73	5289.65
3	4	172.73	100	100	7273
3	1	107.73	20	400	35092
Total congestion charge					49637.40

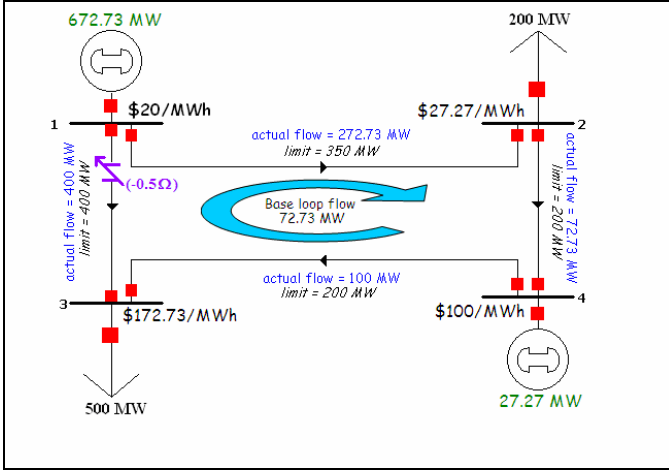


Fig. 9. System with TCSC in line 1-3 ($X_{1,3}=0.5\Omega$)

In short, loop flows always occur in interconnected loop networks. As long as there is a constrained line, nodes within the loop could experience volatile nodal prices thus creating different congestion charges and FTR allocations. It is also observed that small adjustment of TCSC can hugely impact the FTR prices in LMP market. Other devices like phase shifters or DC lines could impact the FTR market in a similar way.

VI. CONCLUSIONS

This paper demonstrated LMP volatilities, termed as spring washer in NZEM, during congestion in network with a loop. The spring washer effect was shown to depend on the network parameters and loop flow patterns. Scenarios were also illustrated showing the operational impacts of loop flow control devices like TCSC on FTR congestion rentals. These situations could potentially occur in future trading periods of markets that are restructuring to incorporate SMD guidelines. Hence understanding loop flow patterns, both major and minor, should be helpful in bidding for FTR auctions. Alternatively, procedures should be in place to ensure that operation of these devices in real-time market is not aimed exclusively to favor certain market participants.

VII. APPENDIX

Appendix I: Pricing model for a N-bus network [14]

Primal Problem:

The objective function is to minimize the total generation cost:

$$\text{Min } Z = \sum_{(b,i)} c_{bip} P_{gbi} \quad (5)$$

where c_{bip} = energy offer price of block b by generator i ,
\$/MWh

P_{gbi} = Quantity offered for energy of block b by
generator i , MW

Subject to the following constraints:

$$\text{Power balance: } P_{gi} - P_{di} = \sum_{j \in Ni} P_{ij} \quad : \lambda_i^P \quad (6)$$

where P_{di} = demand at bus i during normal conditions, MW

P_{gi} = dispatch generation at bus i to satisfy the demand

requirements, MW

$j \in Ni$ = All nodes connected to bus i

Cleared energy offer blocks satisfied by all dispatched generator i :

$$\sum_{b,i} P_{gbi} - P_{gi} = 0 \quad : \sigma_i \quad (7)$$

Maximum and minimum energy offered block sizes:

$$-P_{gbi} \geq -P_{gbi}^{\max} \quad : \gamma_{bi}^+ \quad (8)$$

$$P_{gbi} \geq 0 \quad : \gamma_{bi}^- \quad (9)$$

Demand requirement specified at a bus:

$$P_{di} = P_{di}^{set} \quad : \beta_i \quad (10)$$

Power flows in a line between node i and j during normal conditions:

$$P_{ij} = B_{ij} (a_i - a_j) \quad : \tau_{ij}; j > i \quad (11)$$

$$P_{ji} = -P_{ij} \quad : \psi_{ij}; j > i \quad (12)$$

where B_{ij} = admittance in pu

a_i = Bus angle in radian

Swing bus angle for a slack bus during normal condition:

$$a_i = 0 \quad : \pi_i; i = 1 \quad (13)$$

Maximum and minimum limits for power flow in lines during normal condition:

$$-P_{ij} \geq -P_{ij}^{\max} \quad : \phi_{ij}^+; j > i \quad (14)$$

$$P_{ij} \geq P_{ij}^{\min} \quad : \phi_{ij}^-; j > i \quad (15)$$

Positive variables: P_{gi} , P_{ij}

The multipliers located at far right side associated with each of the inequality constraints are non-negative. These are called marginal price.

Dual Problem:

$$\left. \begin{aligned} L = & \sum_{b,i} c_{bip} P_{gbi} + \lambda_i^P \left(\sum_{j \in Ni} P_{ij} - P_{gi} + P_{di} \right) \\ & + \sum_i \sigma_i (P_{gi} - \sum_{b,i} P_{gbi}) + \sum_{b,i} \gamma_{bi}^+ (P_{gbi} - P_{gbi}^{\max}) \\ & - \sum_{b,i} \gamma_{bi}^- P_{gbi} + \sum_i \beta_i (P_{di}^{set} - P_{di}) \\ & + \sum_{ij; j>i} \tau_{ij} [B_{ij} (a_i - a_j) - P_{ij}] - \sum_{ij; j>i} \psi_{ij} (P_{ij} + P_{ji}) \\ & - \sum_i \pi_i (a_i) + \sum_{ij; j>i} \phi_{ij}^+ (P_{ij} - P_{ij}^{\max}) - \sum_{ij; j>i} \phi_{ij}^- (P_{ij}^{\min} - P_{ij}) \end{aligned} \right\} \quad (16)$$

The dual constraints are:

$$c_{bip} - \sigma_i + \gamma_{bi} = 0 \quad : P_{gbi} \quad (17)$$

$$-\lambda_i^P + \sigma_i = 0 \quad : P_{gi} \quad (18)$$

$$\lambda_i^P - \beta_i = 0 \quad : P_{di} \quad (19)$$

$$-\tau_{ij} + \lambda_i^P + \phi_{ij} - \psi_{ji} = 0 \quad : P_{ij}; j > i \quad (20)$$

$$\lambda_j^P - \psi_{ji} = 0 \quad : P_{ji} \quad (21)$$

$$\sum_{j>i} \tau_{ij} B_{ij} - \sum_{j<i} \tau_{ji} B_{ji} = 0 \quad : a_i; i \neq 1 \quad (22)$$

$$\pi_1 = 0 \quad : a_1 \quad (23)$$

$$\gamma_{bi} = \gamma_{bi}^+ - \gamma_{bi}^-; \quad \phi_{ij} = \phi_{ij}^+ - \phi_{ij}^- \quad (24)$$

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IX. BIOGRAPHIES

Chin Yit Choo graduated from University of Auckland with Bachelors Degree in Electrical and Computer Engineering in 2004. She is currently pursuing her ME degree in Power System Engineering. She also had gained some working experience with ABB New Zealand Limited under Automation Services Group. Her current research areas involve load modeling and electricity markets. Ms. Chin is a student member of the IEEE.

Nirmal-Kumar C Nair received his BE in E.E. from M.S. University, Baroda, India in 1990 and ME in E.E with specialization of High Voltage Engineering from Indian Institute of Science, Bangalore, India in 1998. He received his Ph.D. in E.E. from Texas A&M University, College Station, USA in 2004. He has held several professional, teaching and research positions. Presently, he is a Lecturer at the Department of Electrical & Computer Engineering in University of Auckland, New Zealand. His current interest involves power system analysis & optimization in the context of electricity markets and integration issues of DG/renewable sources into bulk power system. Dr. Nair is a member of the IEEE.

Bhujanga B. Chakrabarti is a Senior Development Engineer with System Operations Group in Transpower New Zealand Limited. He obtained his BEE (Distinction) degree from Jadavpur University, India, MS from Northern Illinois University, MEE from University of Newcastle, Australia and Ph.D. from University of Canterbury, New Zealand in 2004. His areas of interest are power system analysis, control, planning and electricity pricing. Dr. Chakrabarti is a member of the IEEE.