Modeling of Wind Generation Fluctuations in a Dispatch Model

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Abstract--**The speed of wind at the different wind generator sites fluctuates due to weather variations over time. As a result, the wind generations may fluctuate over a wide range. The fluctuations and uncertainty are presently covered, in the New Zealand electricity market (NZEM), by the reserves originally offered for the protection of larger risk units in the system and frequency regulating reserve with no cost to the wind generators. In fact, smaller units benefit from the magnitude of reserves determined by the largest risk units. The frequency of outage of wind generators and variability of wind generation outputs is quite high compared to other smaller units and these frequently need a considerable amount of reserves to cover the generation fluctuations. The wind generators, therefore, "free ride" on other reserve providers adding cost to the system but not necessarily sharing them in an equitable manner. This issue has not been addressed so far.**

We develop a dispatch optimization model that recognizes the additional risk imposed by the wind generators. The model is based on DC-OPF, driven by generator risk contingencies, security margins at the load buses to cover the large variation of load, and security margins at the wind generator buses to account for the uncertainty of wind generations. The margins are maintained using reserves. The margin required at each wind generator bus is an input to the model and can be estimated statistically from its past performance.

The main thrust of this paper is to examine the effect of security margin constraints at the wind generator buses and also at the load buses on the nodal spot prices. The paper demonstrates that the generation, and demand prices at a bus are different in the presence of binding security margin constraints. Generation prices at the wind generator buses get reduced in the presence of binding security margin constraints at these buses.

Index terms--**Contingencies, Dual analysis, Generator risk, Linear program, Nodal spot price, Reserves, Security margin, Wind generator.**

I. INTRODUCTION

THE lack of reliable power supply from wind generation is well recognized and studies around the world have demonstrated the effective availability of wind generations during peak period can indeed be very low – figures as low as 20% of installed capacity have been reported in the literature. While this is true, there have not been a formal procedure or model in place that relates this lack of reliability to a market based pricing mechanism even in advanced second generation electricity markets that have elaborate reserve pricing mechanism. This paper puts forward a concept to formally recognise the lack of reliability of wind generators for critical periods, e.g., during the contingency state. It also integrates the concept to a market clearing model for energy and reserve co-optimisation and explains the pricing implications for unreliable generators.

The literature on electricity pricing has focused on many practical aspects of the power system e.g., system security [3, 4] load frequency control [5] reactive power and voltage constraints [6, 7], reserve modeling [9, 10]. The effect for maintaining voltage stability margin on spot pricing has been discussed [8] and the effect of maintaining multiple security margins at the load buses using reserves is discussed in [11]. The use of spot pricing to reflect the marginal cost of generation, transmission capacity and system security constraints for real power is widely accepted today. Sophisticated real-time pricing of both energy and ancillary services including contingency and frequency control reserve have been operating in several markets worldwide including the New Zealand Electricity Market [13]. In the NZEM, sufficient reserve generation in the system is maintained so that these reserves can be called on and used to cover the outage of the "risk generators". The risk contingencies are well defined, and the amount of reserve needed to cover such contingencies is determined dynamically using energy and reserve co-optimisation.

Under the current reserve pricing arrangements in the NZEM, the generators that cause under frequency events are penalised but no such penalty exists if a load fluctuates causing a change in frequency that needs reserve to address the issue. The effect of such load variation is modeled by maintaining a security margin at the load buses using reserve. The margin could be expressed as a percent of initial load. The effect of wind generation variation is also modeled by maintaining a margin at the wind generator buses using reserve. The effect of maintaining such margins is captured directly in nodal spot prices.

Though different kinds of reserves are required to manage system frequency, we consider, without loss of generality, only one type of reserve in the model.

Generator risk, reserve and security margin modeling are discussed in section 2. Section 3 presents a linear primal dispatch and pricing model, driven by generator risk constraints, and security margin constraints at the load buses and at the wind generator buses. The section also examines the effect of such margins on the nodal prices for generation, reserve and demand at the different buses by performing dual

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analysis. A simple numerical example is discussed in section 4 followed by conclusions, acknowledgements, and references in sections 5, 6, and 7, respectively.

II. RISK AND RESERVE MODELLING

A. Generator Risk Contingency Modeling

It is necessary to have some reserve generation in the system so that these reserves can be called on and used to cover the outage of the so-called `risk generators'. We consider that the requirement of reserve is set by a `risk unit' or by a pre-defined estimated risk (constant). For a risk unit *u*, both energy and reserve produced by it are lost when the unit trips and the available reserve from the other units must cover the power output of the 'risk unit'. The reserve requirement differs at different system conditions even for the outage of the same unit with same dispatched generation. We introduce Risk Adjustment Factor called "RAF" to determine the total requirement of reserve at different system operating conditions [10].

The total requirement of reserve becomes,

$$\sum_i R_i \ge RAF_u.P_u + R_u;\ \ u \in Risk\ units \qquad (2.1)$$

One such constraint is required for each `risk unit'. The model dynamically sets the risk. The reserve cleared should satisfy each of these constraints so that cleared reserve can cover any of these contingencies. The solution algorithm chooses the best set of units to have as the risk, optimally trading the risk off against the cost of providing reserve and the incentives to generate. Only the constraints for the units defining the risk may be binding under most normal circumstances.

B. Reserve Modeling

Reserve characteristics, in the dispatch model, could be represented in different ways depending on the market structure, e.g., whether unit commitment (UC), automatic generation control (AGC) and economic dispatch (ED) functions are available. One such characteristic is discussed in [12] which necessitate the introduction of integer to model the various reserves. This needs Mixed Integer Programming (MIP) solver to solve the problem. We, however, present a reserve characteristic that does not require MIP solver to solve the problem. Energy and reserves utilise the same plant to provide different amounts of energy and reserve determined by the trade off function of these two products. The reserve characteristic for a self committed unit looks like an "inverted bath tub [9, 10]. The New Zealand market uses this kind of reserve characteristic [13].

The constraints that describe reserve model for unit *i* are:

Proportional constraint

$$R_i \le b_i.P_i \qquad (2.2)$$

This implies that reserve cleared cannot be greater than a predefined proportion (b_i) of the total generation cleared (P_i) for unit *i*.

Reserve upper bound constraint

$$R_i \le R_i^{\max} \qquad (2.3)$$

Generator joint energy and reserve capacity constraint

$$P_i + z_{u,i}.R_i \le P_i^{cap} \qquad (2.4)$$

Generation upper and lower bound constraints (2.5)

$$\left.\begin{array}{l} P_i \le P_i^{\max} \\ P_i \ge 0 \end{array}\right\} \qquad (2.5)$$

Where

P_i = Cleared generation of generator, *i*

$Z_{u,i}$ = Capacity utilisation factor for a generator *i's* reserve

R_i = Cleared Reserve of generator, *i*

P_i^{max} = Generator's maximum capacity for reserve purpose

For simplicity, we assume zu =1 and RAF =1 for all units in the model described in the next section.

C. Multiple Security Margin Contingency

We define the Security Margin (SM) as a load margin in terms of MW from the current operating point. In order to preserve security, it may be necessary to maintain a different security margin at different buses or in different regions. The objective of the security constraint is to secure the operating point within a pre-defined margin should such a loading condition occur due to some contingency, c. Here we recast the problem as a contingency constrained dispatch in which the critical issue is to be able to meet the load pattern which could emerge as a result of each contingency, given the resources available under that contingency condition. First, the SM at each bus *i* during contingency c is defined as [11,14]:

$$k_{i,c} = \frac{P_{di,c} - P_{di}}{P_{di,c}};\forall c\,, \qquad (2.6)$$

$P_{di,c}$ Demand at bus *i* during contingency condition, c.

P_{di} Demand at bus *i* during normal condition.

$k_{i,c}$ Security Margin at bus i during contingency condition, c.

Rewriting (2.6), we get,

$$P_{di,c} = \frac{P_{di}}{1-k_{i,c}};\forall i,\forall c \qquad (2.7)$$

We require the total reserve to be able to meet each contingency c, i.e.,

$$\sum_i R_i \ge \sum_i P_{di,c} - \sum_i P_{di}\,;\forall c \qquad (2.8)$$

Thus using (2.7),

$$\sum_i R_i \ge \sum_i \frac{k_{ic}}{1-k_{i,c}} P_{di}\,;\forall c \qquad (2.9)$$

The total reserve required is thus dependent on the bus loads and the margin required during the contingencies.

D. Security Margin At The Wind Generator (WG) Bus

The security margin at the wind generator buses can be thought of as being related to uncertainty and fluctuations in wind generations. The wind generation variability could be different at different buses and the margin at the wind generator buses is assumed to be a function that may be statistically estimated using data of their past performance. The margin in the model is maintained using reserve. The margin, as stated above, is an input to the model and estimated statistically, off line. We discuss the impact of this margin on the spot prices. It may be noted that, in practice, it might be maintained using regulating reserve and should be suitably modeled.

First, the security margin at each WG bus i during contingency c is defined as:

$$k_{gi,c} = \frac{P_{gi} - P_{gi,c}}{P_{gi}}; \forall c, i \in \text{ wind generators} \qquad (2.10)$$

P_{gi} Generation at bus i during normal condition.

$P_{gi,c}$ Generation at bus i during contingency c

$k_{gi,c}$ Security Margin at the wind generator bus i. It could range between 0 and 1 both inclusive, and represents the reduction in nett generation available from a wind generator in the contingency state. The buses i where $k_{gi,c}$ is non-zero are included in the set of WG buses, a sub set of generator buses. Rewriting (2.10), we get,

$$P_{gi,c} = P_{gi}(1 - k_{gi,c}); \forall i, \forall c \qquad (2.11)$$

E. Arrangement of Reserve

The reserve market is assumed to trade reserve in addition to energy for each trading period. The reserve providers would be paid a "stand-by" fee irrespective of whether a contingency actually occurs or not. They may receive little or nothing extra during a contingency. The market operator will co-optimise energy and the reserve for each dispatch period. We restrict our analysis only for a single dispatch period.

III. MODEL

A. Primal Problem

We consider a case where multiple generators are supplying multiple load buses and each generator has multiple offers for energy and multiple offers for reserves. The reserves should be enough to cover the generator contingency risk as well as maintain the required margins at the load and wind generator buses. The model is driven by both the multiple generator contingency constraints and the multiple security margin contingency constraints. The objective of this study is to examine the effects of these constraints on the nodal prices for energy and reserves. We present the model in this section. The duals, or shadow prices (multipliers) associated with the constraints are shown on the right hand side of each constraint.

The objective function is to minimise the combined generation (energy) and reserve cost:

$$\text{Min } Z = \sum_{(b,i)} (c_{bip}.P_{bi} + c_{bir}.R_{bi}) \qquad (3.1)$$

Where,

c_{bip} = Energy offer price of block b by generator i, \$/MW

P_{bi} = Quantity offered for energy of block b by generator i, MW

c_{bir} = Reserve offer price of block b by generator i, \$/MW

R_{bi} = Quantity offered for reserve of block b by generator i, MW

Note that each bus can have both load and generation so that both generation and demand prices at the bus can be determined even if one is absent. In our model, though, each bus has either generation or demand, not both and indexed by '*i*'. Also we have indexed both energy and reserve blocks by '*b*'.

The following constraints are applied into the model.

We define a set Ni ={j: there is a line between i and j}. Power flows in the line between i and j during normal and contingency conditions are given by (3.2) to (3.5).

$$P_{ij} = B_{ij}(a_i - a_j) : \tau_{ij}; (\forall i, j \in Ni; j > i) \qquad (3.2)$$

$$P_{ij,c} = B_{ij}(a_{i,c} - a_{j,c}) : \tau_{ij,c}; (\forall i, j \in Ni; j > i); \forall c \qquad (3.3)$$

$$P_{ji} = -P_{ij} : \psi_{ij}; (\forall i, j \in Ni; j > i) \qquad (3.4)$$

$$P_{ji,c} = -P_{ij,c} : \psi_{ij,c}; (\forall i, j \in Ni; j > i); \forall c \qquad (3.5)$$

B_{ij} = Line susceptance between i and j, in pu
a_i, a_{ic} = Bus angles at bus i, in radian

The power balance at each bus must be preserved during normal and contingency conditions. Given the conventions defined above, these are given by (3.6) and (3.7).

$$P_{gi} - P_{di} = \sum_{j \in Ni} P_{ij} : \lambda_i; \forall i \qquad (3.6)$$

$$P_{gi,c} + R_{i,c} - P_{di,c} = \sum_{j \in Ni} P_{ij,c} : \lambda_{i,c}; \forall i; \forall c \qquad (3.7)$$

$j \in$ Ni All nodes connected to bus i, as defined earlier
$P_{di,c}$ Demand at bus i during contingency c
$P_{d,i}$ Demand at bus i during normal conditions
$R_{i,c}$ Reserve response from unit i during contingency c

Generation at bus i during contingency c is given by (3.8).

$$P_{gi,c} = P_{gi}(1 - k_{gi,c}) : \xi_{gi,c}; \forall i, \forall c \qquad (3.8)$$

Swing bus (s) angles during normal and contingency conditions are set at zero as shown in (3.9) and (3.10).

$$a_i = 0 : \pi_i; i = s \qquad (3.9)$$

$$a_{i,c} = 0 : \pi_{i,c} ; i = s ; \forall c \quad (3.10)$$

The demand in normal conditions is given by a pre-set level as shown in (3.11). The relationship between demand in normal and contingency conditions is given by (3.12).

$$P_{di} = P_{di}^{set} : \beta_i ; \forall i \quad (3.11)$$

$$P_{di,c} = \frac{P_{di}}{1 - k_{i,c}} : \beta_{i,c} ; \forall i ; \forall c \quad (3.12)$$

where, $k_{i.c}$= Margin at bus *i* during contingency, *c*.

Maximum and minimum limits on power flow in lines during the normal and contingency conditions are given by (3.13) to (3.16).

$$-P_{ij} \ge -P_{ij}^{\max} : \phi_{ij}^{+} ; (\forall i, j \in N_i ; j > i) \quad (3.13)$$

$$P_{ij} \ge P_{ij}^{min} : \phi_{ij}^{-} ; (\forall i, j \in N_i ; j > i) \quad (3.14)$$

$$-P_{ij,c} \ge -P_{ij,c}^{\max} : \phi_{ij,c}^{+} ; (\forall i, j \in N_i ; j > i) ; \forall c \quad (3.15)$$

$$P_{ij,c} \ge P_{ij,c}^{\min} : \phi_{ij,c}^{-} ; (\forall i, j \in N_i ; j > i) ; \forall c \quad (3.16)$$

The limits could be thermal or derived from the other operating conditions. The limits are only applied to flows in the conventional “Forward direction”, but given (3.4) and (3.5) they are equally effective with respect to flows in either direction. The limits in the contingency condition may be set at a higher value, including contingency overload.

The sum of cleared generation offer blocks of generator *i* must be equal to the dispatched generation of *i* as shown in (3.17). Each cleared generation offer block is constrained by its maximum and minimum level as shown in and (3.18) and (3.19).

$$\sum_b P_{gbi} - P_{gi} = 0 : \sigma_i ; \forall i \quad (3.17)$$

$$-P_{gbi} \ge -P_{gbi}^{\max} : \gamma_{bi}^{+} ; \forall b \quad (3.18)$$

$$P_{gbi} \ge 0 : \gamma_{bi}^{-} ; \forall b \quad (3.19)$$

The sum of cleared reserve offer blocks of generator *i* must be equal to the dispatched reserve R_i as shown in (3.20). Reserve response from unit *i* in contingency *c* ($R_{i,c}$) must be less than or equal to the dispatched reserve from unit *i* (R_i) as shown in (3.21). Each cleared reserve offer block is constrained by its maximum and minimum level as shown in (3.22) and (3.23).

$$\sum_b R_{bi} - R_i = 0 : \mu_i ; \forall i \quad (3.20)$$

$$R_i - R_{i,c} \ge 0 : \rho_{i,c}^{m} ; \forall i ; \forall c \quad (3.21)$$

$$-R_{bi} \ge -R_{bi}^{\max} : \varepsilon_{bi}^{+} ; \forall b \quad (3.22)$$

$$R_{bi} \ge 0 : \varepsilon_{bi}^{-} ; \forall b \quad (3.23)$$

The joint generation and reserve on any machine should lie within the maximum and minimum capacities of the machine as shown in (3.24) and (3.25).

$$-P_{gi} - R_i \ge -P_{gi}^{cap\_h} : \upsilon_i^{+} ; \forall i \quad (3.24)$$

$$P_{gi} + R_i \ge P_{gi}^{cap\_l} : \upsilon_i^{-} ; \forall i \quad (3.25)$$

Dispatched reserve can not exceed some proportion (x_i) of dispatched generation level as shown in (3.26).

$$-R_i \ge -x_i P_{gi} : \theta_i ; \forall i \quad (3.26)$$

The reserve from all generators must be able to meet any generator risk contingency as shown in (3.27) and (3.28).

$$R^g \ge P_{gu} + R_u : \rho_u^g ; \forall u \quad (3.27)$$

$$\sum_i R_i \ge R^g : \rho \quad (3.28)$$

R^g = Total reserve required to cover the generator risk contingencies

u = Risk generators as defined before.

The multipliers associated with the inequality constraints are non-negative.

Though we discussed about the effect of margins at the wind generator buses, the model is able to handle the margin at any generator buses, particularly at the non-compliant generator buses.

B. Dual Problem

First, construct Dual constraints from the first order optimality Karush-Kuhn-Tucker (KKT) condition. We show the associated primal variables on the right hand side of the dual constraints. The objective function of the dual problem is a Lagrangian type function formed from the both equality and inequality constraints and complementary slackness conditions. This function represents the total cost to the system. As long as the prices satisfy both dual constraints and complementary slackness conditions, the dual objective function is redundant (assuming unique dual solution). This allows us to use any arbitrary dual objective function [15]. Note that the dual objective function has the same value as the primal objective function at the optimal solution. The set of linear equations given in the dual problem needs to be solved simultaneously in order to obtain mutually consistent prices of generation, reserve and demand. In these equations, some variables are fixed (c_{bip}, c_{bir}) and some (prices) are variables such as $\gamma_{bi}, \upsilon_i, \rho_u^g, \lambda_i, \lambda_{i,c}, \beta_i, \beta_{i,c}$ etc. These variables take values that they need to produce consistent prices. The set of dual constraints are shown in (3.29).

$$
\begin{aligned}
&c_{bip} - \sigma_i + \gamma_{bi} = 0 \quad :\mathrm{P}_{\mathrm{gbi}}, \forall b \\
&-\lambda_i + \sum_c \xi_{gi,c}(1 - k_{gi,c}) + \sigma_i + \upsilon_i - x_i\theta_i + \\
&\rho_i^g.\delta_{pi} = 0 \; :P_{gi}, \forall i \\
&\delta_{pi} = 1 \text{ for binding risk units; otherwise } \delta_{pi} = 0 \\
&c_{bir} - \mu_i + \varepsilon_{bi} = 0 \quad :R_{bi}; \forall b \\
&\rho_i^g.\delta_{ri} - \sum_c \rho_{i,c}^m - \rho + \mu_i + \upsilon_i + \theta_i = 0 \quad :R_i \\
&\delta_{ri} = 1 \text{ for binding risk units; otherwise } \delta_{ri} = 0 \\
&-\lambda_{i,c} - \xi_{gi,c} = 0 : P_{gi,c}, \forall i \\
&-\sum_u \rho_u^g + \rho = 0 \quad :R^g; \forall i \qquad\qquad (3.29) \\
&-\lambda_{i,c} + \rho_{i,c}^m = 0 \quad :R_{i,c}; \forall i \\
&\lambda_i + \frac{\beta_{i,c}}{1-k} - \beta_i = 0 \quad :P_{di}; \forall i \\
&\lambda_{i,c} - \beta_{i,c} = 0 : P_{di,c}; \forall i \\
&-\tau_{ij} + \lambda_i + \phi_{ij} - \psi_{ji} = 0 : P_{ij}; (\forall i, j \in N_i, j > i) \\
&\text{where } \; \phi_{ij} = \phi_{ij}^+ - \phi_{ij}^- \\
&\gamma_{bi} = \gamma_{bi}^+ - \gamma_{bi}^-; \quad \varepsilon_{bi} = \varepsilon_{bi}^+ - \varepsilon_{bi}^-; \upsilon_i = \upsilon_i^+ - \upsilon_i^- \\
&\lambda_j - \psi_{ji} = 0 : P_{ji} \\
&-\tau_{ij,c} + \lambda_{i,c} + \phi_{ij,c} - \psi_{ji,c} = 0 : P_{ij,c} \\
&\lambda_{j,c} - \psi_{ji,c} = 0 : P_{ji,c} \\
&\sum_{j \in N_i, j > i} \tau_{ij} B_{ij} - \sum_{j \in N_i, j < i} \tau_{ji} B_{ji} = 0 : a_i, i \neq s \\
&\pi_s = 0 : a_s, i = s \\
&\sum_{j \in N_i, j > i} \tau_{ij,c} B_{ij} - \sum_{j \in N_i, j < i} \tau_{ji,c} B_{ji} = 0 : a_{i,c}, i \neq s \\
&\pi_{s,c} = 0 : a_{s,c}, i = s
\end{aligned}
$$

C. Generation Price Relationship

Using the dual constraint corresponding to primal variable, Pg_i,

$$\lambda_i = c_{bip} + \gamma_{bi} + \upsilon_i - x_i\theta_i + \rho_i^g.\delta_{pi} - \sum_c \lambda_{i,c} + \sum_c \lambda_{i,c} k_{gi,c}; \forall i \quad (3.30)$$

Generation price at bus *i* is defined as sum of bus lambda, λ_i, and payment and charges made for its contribution to meeting load and margins at the wind generator buses in the contingency condition, and is given by (3.31).

$$\lambda_i + \sum_c \lambda_{i,c} - \sum_c \lambda_{i,c}.k_{gi,c} = c_{bip} + \gamma_{bi} + \upsilon_i - x_i\theta_i + \rho_i^g.\delta_{pi} \quad (3.31)$$

The difference between generator price and the generator offer price is explained by the cost of its own reserve, and the (opportunity) "cost" of other binding resources. The above generation price equation corresponds to:

1. The cost of block of generation

This is the nett cost due to generation C_{bip}, plus the opportunity cost of block size limits, γ_{bi} ,if binding.

2. The cost due to joint capacity constraint for individual unit (υ_i), if binding

The joint capacity constraint is represented as a line with negative slope as shown in (3.24). The generation plus reserve is constrained by the capacity limit of the generator. Each MW increase of generation (energy) reduces the reserve response by 1 MW or vice versa.

3. Minus, the cost due to the proportional reserve constraint for each individual unit, $x_i\theta_i$, if binding

4. (Plus) The cost due to generator risk contingency ρ_i^g , if binding

The multiplier ρ_i^g is associated with the generator risk constraint. If the risk generator *i* sets the risk, i.e., *i* th risk reserve constraint is binding, then $\delta_{pi} = 1$ and ρ_i^g >0. Thus ρ_i^g is the cost of reserve to cover its own generation and the generator should be charged for it rather than being paid.

5. The nett cost of bus *i*'s generation as a contribution to meeting margins during the different contingencies ($\sum_c \lambda_{i,c} - \sum_c \lambda_{i,c}.k_{gi,c}$)

The multiplier $\lambda_{i,c}$ is associated with the power balance constraint for bus *i* during the contingency condition c. Notice that P_{gbi} is present in the power balance constraints for both normal and contingency conditions. The generation in the power balance equation during the contingency condition is expressed in the form of (P_{gi} (1-K_{gi})+ R_i). Any increase in generation dispatch during normal conditions implies an equivalent increase in output for meeting the increased demand during each contingency. This is rewarded by $\rho_{i,c}^m = \lambda_{i,c}$,where $\rho_{i,c}^m$ is the multiplier associated with reserve dispatch during the contingency, *c*. But there is another component, $-\lambda_{i,c}k_{gi,c}$ which is the cost of reserve for maintaining margin at its own (wind generator) bus. Thus the effective price to be paid to the generator is not λ_i but $\lambda_i + \sum_c \lambda_{i,c} - \sum_c \lambda_{i,c}.k_{gi,c}$. But it is essential to note that, for most *i*, the λ_i's and γ_{bi}'s are not determined by this generator's offer. They are determined by the marginal generator's offers. Other multipliers for this generator then take whatever values they have to in order to make up the difference between these market clearing prices and their offers.

D. Reserve Price Relationship

The reserve price at bus *i* is given by the multipliers associated with the constraints defining reserve dispatch to meet the security margins and the generator risk reserve constraints. The reserve price relationship at bus *i,* is given by (3.32).

$$\left.\begin{aligned} &[\sum_{c}\lambda_{i,c}+\rho]-\rho_i^g.\delta_{ri}=c_{bir}+\varepsilon_{bi}+\upsilon_i+\theta_i \\ &\rho=\sum_{u\in u^b}\rho_u^g \quad ; \quad u^b=\text{Binding risk units} \end{aligned}\right\} \quad (3.32)$$

The reserve price at bus *i* corresponds to:

1. The offer cost of a reserve block
2. The opportunity cost due to that reserve block's size limits, if binding.
3. The cost due to the joint capacity constraint for each individual unit, if binding
4. The cost due to the proportional reserve constraint for each individual unit, if binding.

E. Demand Price Relationship

The multiplier associated with the demand constraint at bus *i* in primal problem defines the demand price there. The demand price relationship at bus *i,* is given by (3.33). The demand price is driven by λ_i and $\lambda_{i,c}$ from the supply side only and no trade off from the demand side is considered because we have not considered demand bids.

$$\beta_i=\lambda_i+\sum_{c}\frac{\lambda_{i,c}}{1-k_{i,c}} \qquad (3.33)$$

β_i = Multiplier associated with the demand constraint at bus *i*, which defines the demand price there.

Thus the demand price, in the presence of a binding security margin constraint, indicates not only the marginal cost of meeting the demand, but also the additional cost of reserve that is required for maintaining margin.

IV. NUMERICAL EXAMPLES

A 10 bus loss-less system is considered as shown in Fig. 1. Buses 1 to 5 are generator buses and 6 to 10 are demand buses. Each line and each transformer has an admittance (B) of 50 pu and 500 pu respectively. The demands under the normal condition, and the security margins at the load buses during different contingencies are shown in Table I. The generation and reserve offers and their characteristics are shown in Table II. The second and third columns of the table show the energy and reserve prices and the last three columns show the energy, capacity and reserve limit of the generators. The minimum limits are zero for all generators. The generators at buses 2, and 3 are wind generators and the margin at these buses during different contingencies are shown in Table III.

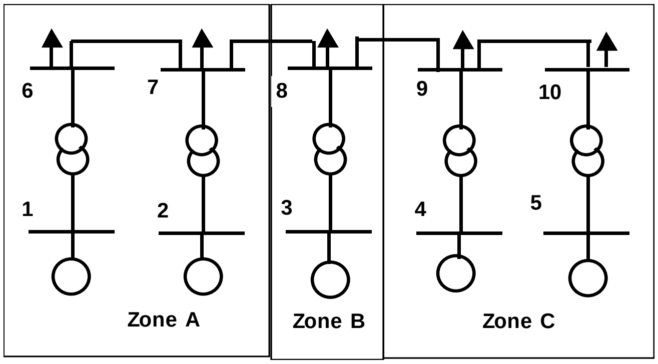

Fig. 1. A 10 bus network

TABLE I
DEMAND AND SECURITY MARGINS

Load Bus	C1	C2	C3	Load MW
6	0.3	0.3	0.3	125
7	0.3	0.2	0	50
8	0.2	0.2	0.3	100
9	0.15	0.2	0.2	150
10	0.2	0.2	0.3	200

TABLE II
GENERATOR OFFERS AND DATA

Gen Bus	Pg, $/MW	Rg, $/MW	Pg-max, MW	cap, MW	Rg_max, MW
1	35	15	300	300	200
2	20	15	100	100	0
3	25	15	100	100	0
4	30	15	300	300	200
5	40	15	300	300	200

This analysis is carried out to examine the effect on the nodal prices of security margins and the generator risk constraints. The analysis is conducted in three steps:

Case 1: Only multiple security margins at the load buses are considered.

Case 2: Multiple security margins both at the load buses and the wind generator buses are only considered.

Case 3: Generator risk constraints are considered in addition to case 2 above.

Solutions

- *Case 1*

The problem is solved by BDMLP solver. The λ_i 's during the normal condition for all buses are $20. The contingency C3 is binding for all buses with $\lambda_{i,c3}$= $15; $\forall i$. The generation price for all generators =λ_i+ $\lambda_{i,c3}$= $35/MW.

Reserve price for all generators = $\lambda_{i,c3}$ = $15/MW. Demand prices and payments to be made by the loads are shown in Table IV. The generation prices and payments to be made to the generators are shown in Table V.

TABLE III
SECURITY MARGINS AT THE GENERATOR BUSES

Gen Bus	C1	C2	C3
1	0	0	0
2	0.2	0.2	0.2
3	0.1	0.1	0.1
4	0	0	0
5	0	0	0

TABLE IV
DEMAND PRICES AND PAYMENTS BY LOADS: CASE 1

Bus	Load, MW	Dem Price $/MW	Load pays, $
6	125	41.43	5178.57
7	50	35.00	1750.00
8	100	38.75	3875.00
9	150	38.75	5812.50
10	200	41.43	8285.71
Total	625		24901.79

- Case 2

The λ_i during the normal condition for all buses are $20. The contingency C3 is binding at all buses with $\lambda_{i,c3}$= $15. The demand price remained exactly same as in case 1. The generation price for all generators other than wind generators $=\lambda_i+\lambda_{i,c3}$= $35/MW. The generation price for the wind generators $=\lambda_i+\sum_c \lambda_{i,c}-\sum_c \lambda_{i,c}.k_{gi,c}$; for example, for i = 2, price = 20 + 15 – (15 . (0.2)) = $32 /MW. Reserve price for all generators = $\lambda_{i,c3}$= $15/MW. The generation prices and the payments to the generators are shown in Table VI.

TABLE V
OPTIMAL DISPATCH AND PAYMENTS TO GENERATORS: CASE 1

Bus	PG, MW	Reserve, MW	Gen price, $/MW	Reserve Price, $/MW	Payment to Gen, $
1	125	175	35	15	7000
2	100		35	15	3500
3	100		35	15	3500
4	300		35	15	10500
5	0	26.786	35	15	401.79
Total					24901.79

It is observed that 30 MW of reserve is required to maintain margins at the wind generator buses and its cost is $450. The wind generator must pay for this reserve. Wind generators at buses 2 and 3 are paid less by $\sum_c \lambda_{i,c}.k_{gi,c}$; i.e., $3/MW and $1.5/MW than the other generators in order to recover the reserve costs of (100.$3+100.$1.5=) $450.

TABLE VI
OPTIMAL DISPATCH AND PAYMENTS TO GENERATORS: CASE 2

Bus	PG, MW	Reserve MW	Gen price, $/MW	Res Price, $/MW	Payment to Gen
1	125	175	35	15	7000
2	100		32	15	3200
3	100		33.5	15	3350
4	300		35	15	10500
5	0	56.786	35	15	851.79
Total					24901.79

- Case 3

In this case both security margin constraints (c3) and generator risk constraint for generation 4 are binding. The reserve required to maintain security margins for constraint c3 at the load buses and the WG buses is 231.7 MW. The reserve required to meet the generator risk contingency is also 231.7 MW. The demand prices and payments to be made by the loads are shown in Table VII.

The generation prices are given by $\lambda_i+\sum_c \lambda_{i,c}-\sum_c \lambda_{i,c}.k_{gi,c}-\rho_i^g.\delta_{pi}$. The generator risk constraint for generator 4 is binding with a value of ρ_4^g = $5/MW. The reserve price in this case is given by $\lambda_{i,c3}-\rho_4^g$. The λ_i's during the normal condition for all buses are $25/MW. The contingency C3 is binding at all the buses with $\lambda_{i,c3}$= $10/MW; $\forall i$. The generation price for generators 1 $=\lambda_1+\ \lambda_{1,c3}$= $35/MW. The generation price for generators 2 and 3 $=\lambda_i+\ \lambda_{i,c3}-\lambda_{i,c3}k_{gi,c3}$= $33/MW and $34/MW.

TABLE VII
DEMAND PRICES AND PAYMENTS BY LOADS: CASE 3

Bus	Load, MW	Dem Price $/MW	Load pays, $
6	125	39.29	4910.75
7	50	35.00	1750.00
8	100	37.50	3750.00
9	150	37.50	5625.00
10	200	39.29	7857.20
Total	625		23892.95

The generation price for generator 4 = $\lambda_4+\lambda_{4,c3}-\rho_4^g$ = $30 /MW. Reserve price for all generators = $15/MW. The generation prices and payments to be made to the generators are shown in Table VIII. In this case, total reserve required is 231.786 MW and its total cost is $3476.79. This is shared by wind generators (30 MW, $300), risk generators (231.786@$5=$1158.93), and the loads (201.786 @$10=$2017.86).

TABLE VIII
OPTIMAL DISPATCH AND PAYMENTS TO GENERATORS: CASE 3

Bus	PG, MW	Reserve MW	Gen price, $/MW	Res Price, $/MW	Payment to Gen
1	193.214	31.786	35	15	7239.28
2	100		33	15	3300
3	100		34	15	3400
4	231.786		30	15	6953.58
5	0	200	0	15	3000
Total					23892.86

V. CONCLUSIONS

A dispatch and pricing model based on linear programming technique, driven by both the generator risk constraints and the security margin constraints at the load and generator buses is presented to examine their effect on nodal spot prices. It is observed that the generation and the demand prices at a bus are different in the presence of binding security margin constraints. This is because different amount of reserves are required to maintain different security margins at the different buses. However, the effect of security margins could be zero when the generator risk constraint dominates making the SM constraint redundant.

Generation prices at the wind generator buses get reduced in the presence of binding security margin constraints at these buses. The reduced amount matches with the cost of reserve required for maintaining the margins at the WG buses.

The generation price for the risk generators is further reduced by the cost of reserve required to cover its own generation. The allocation of reserve cost among loads, wind generators and risk generators based on multiplier values of the binding constraints is demonstrated in case 3 of the above example.

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