

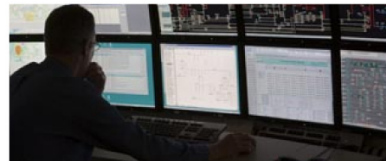
# Multiple Security Constrained Energy Dispatch Model

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- General Nodal Pricing/Dispatch Model: How does it work in Electricity Market
- Modelling of
  - Reserve modelling
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- Primal Dispatch Model co-optimising energy and reserve
- Duality and Dual Formulation
- Derivation of Nodal Prices for Generation, Reserve and the Demand from Dual Analysis
- Numerical Example: With and without transmission congestion

# Issues addressed in Part 2

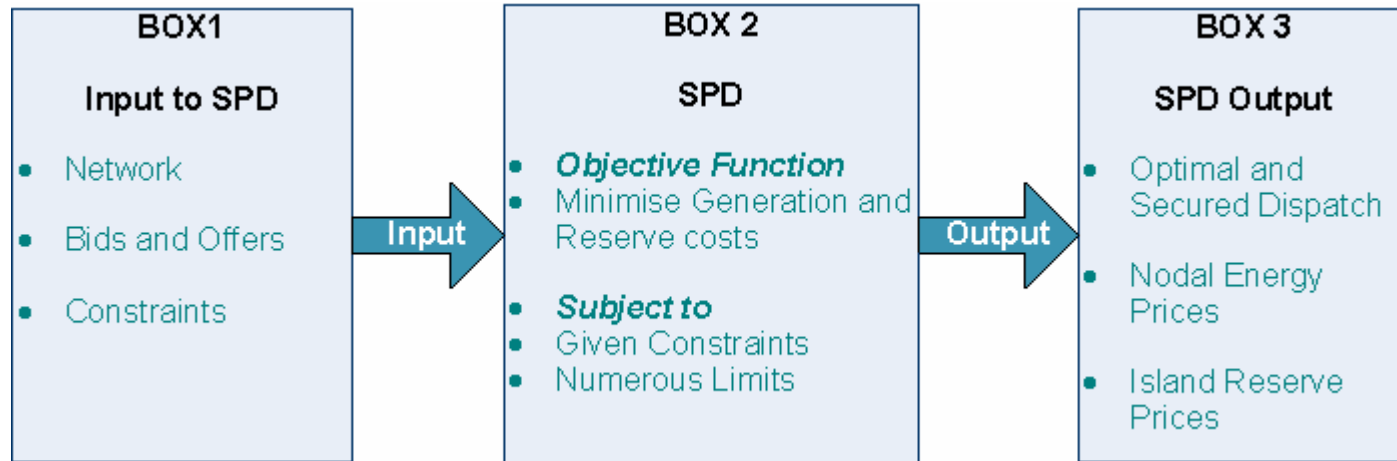
- Multiple Security Margins at the load and the Wind Generator buses
- How do the nodal load prices increase to accommodate the security margins using reserve?
- How to take into account the transmission feasibility?
- How the cost of reserve is realised from loads requiring margins?
- How do the nodal generation prices get reduced to accommodate these variations (margins) using reserve?, and
- How the cost of reserve is transferred from Wind Generators to reserve providers?

# General Nodal Pricing / Dispatch Algorithms

- Find
  - Optimal generation/reserve dispatch, and Nodal spot prices
- To minimise
  - Dispatch cost
- Subject to
  - Meet demand at each “bus” (= node)
  - Meet generator offers
  - Meet power balance at each bus
  - Meet line capacity and other limit constraints
  - Meet N-1 Security and other surrogate constraints
  - Meet unit risk reserve requirements
  - **Meet multiple security margin constraints at different Load Buses**
  - **Meet multiple security margin constraints at Wind Generator Buses**

# SPD in NZEM

SPD is a security constrained DC-OPF based application (works on Linear Program method).

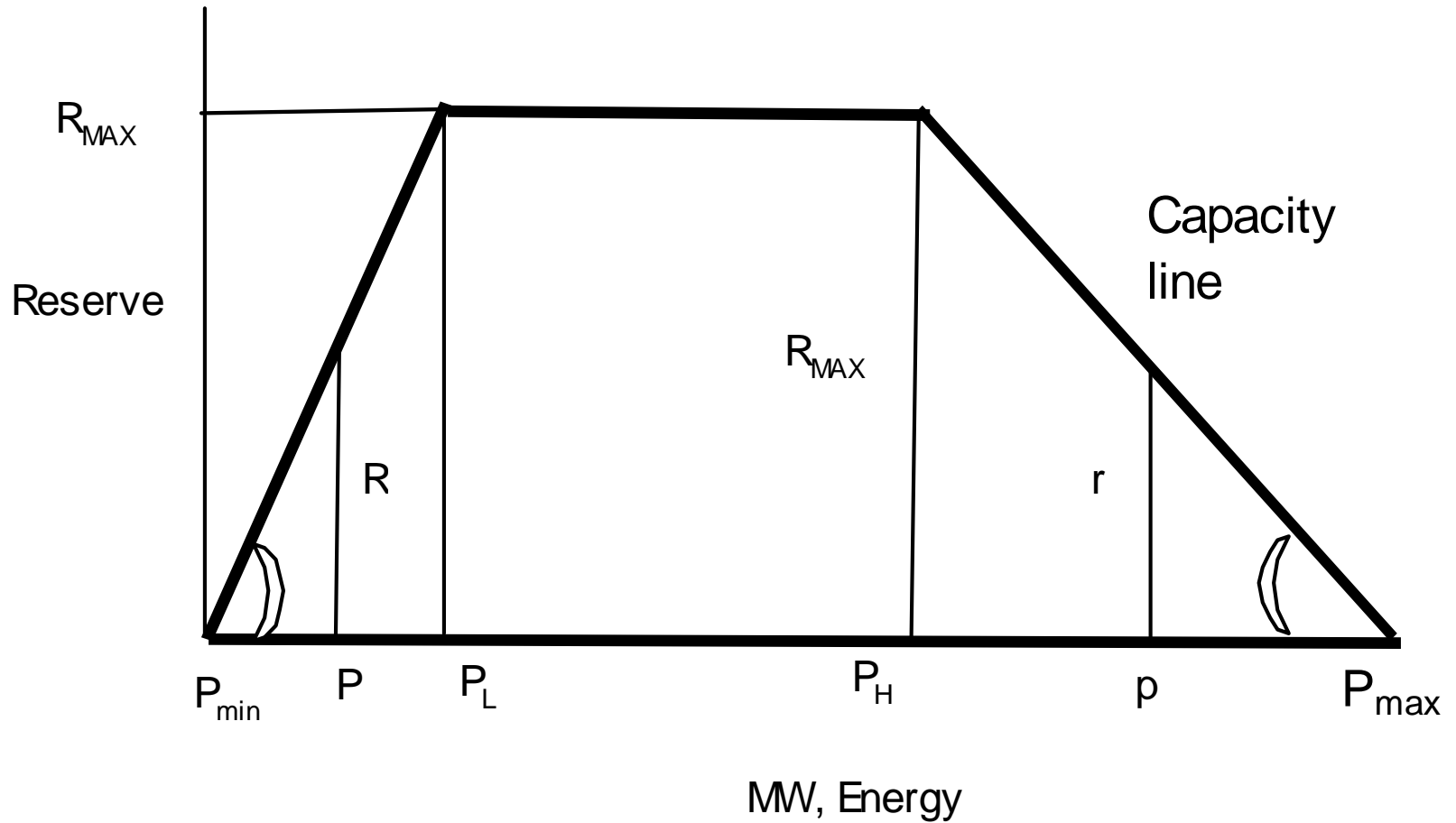


## Major Constraints into SPD

- Bus Injections
- AC & DC Branch Flows
- Branch Losses
- Branch Flow Constraints
- Bus Power Balance Constraints
- Bus Group Generation MW

- Market Node Group Constraints
- Mixed Constraints
- Ramping Constraints
- Risk-Reserve constraints
- N-1 Thermal constraints
- Stability constraints

# Reserve Model



# Reserve constraints

- Proportional constraint

$$R_i \leq x_i \cdot P_i$$

- Reserve upper bound constraint

$$R_i \leq R_i^{\max}$$
$$R_i \geq 0$$

- Generator joint capacity constraint

$$P_i + R_i \leq P_i^{cap}$$

- Generator upper and lower bound constraint

$$P_i \geq 0$$
$$P_i \leq P_i^{\max}$$

# Generator Risk Modelling

- Cover the loss of any of the largest 'Risk units' or Link,
- Size of the risk be optimised,
- Any reserve on a unit cannot be counted when considering the reserve needed to cover that particular unit,

$$\sum_i R_i \geq P_u + R_u; u \in \text{risk units}$$

# Primal DC Model

generation cost + reserve cost

- Objective Function min

$$Z = \sum_{i,b} (C_{b,i,p} P_{gbi} + C_{b,i,r} R_{b,i})$$

$C_{b,i,p}$  = Energy offer price of block  $b$  by generator  $i$  \$/MW

$P_{gbi}$  = Quantity offered for energy of block  $b$  by generator  $i$ , MW

$C_{b,i,r}$  = Reserve offer price of block  $b$  by generator  $i$  \$/MW

$R_{b,i}$  = Quantity offered for reserve of block  $b$  by generator  $i$ , MW

# Model ... contd

- We define a set  $N_i = \{j: \text{there is a line between } i \text{ and } j\}$ . Power flows in the line between  $i$  and  $j$  during normal condition are given by:

$$P_{ij} = B_{ij}(a_i - a_j) : \tau_{ij}; (\forall i, j \in N_i; j > i)$$

$$P_{ji} = -P_{ij} : \psi_{ij}; (\forall i, j \in N_i; j > i)$$

- $B_{ij}$  = Admittance in pu
- $a_j$  = Bus angle in radian

# Model contd...

- Energy Balance

$$P_{gi} - P_{di} = \sum_{j \in Ni} P_{ij} : \lambda_i ; \forall i$$

$P_{di}$  = demand at the load bus  $i$

$\lambda_i$  = Dual variable associated with the power balance equation at bus  $i$

$j \in Ni$  All nodes connected to bus  $i$ , as defined earlier

Note that network Loss is not considered in the model

- Swing Bus Angle  $a_i = 0 : \pi_i ; i = s$

# Model .. contd

- Demand

$$P_{di} = P_{di}^{set} : \beta_i; \forall i$$

- Line Flow Limits

$$-P_{ij} \geq -P_{ij}^{\max} : \phi_{ij}^+; (\forall i, j \in N_i; j > i)$$

$$P_{ij} \geq P_{ij}^{\min} : \phi_{ij}^-; (\forall i, j \in N_i; j > i)$$

- Joint generation and reserve

$$-P_i - R_i \geq -P_i^{cap-h} : u_i, \forall i$$

$$P_i + R_i \geq P_i^{cap-l} : u_i, \forall i$$

# Model ...cont

- Cleared Generation Offer blocks and Gen Dispatch

$$-P_{gbi} \geq -P_{gbi}^{\max} : \gamma_{bi}^+; \forall b, \forall i$$

$$\sum_b P_{gbi} - P_{gi} = 0 : \sigma_i; \forall i$$

- Cleared Reserve Offer blocks and Reserve Dispatch

$$P_{gbi} \geq 0 : \gamma_{bi}^-; \forall b, \forall i$$

$$\sum_b R_{bi} - R_i = 0 : \mu_i; \forall i$$

$$-R_{bi} \geq -R_{bi}^{\max} : \varepsilon_{bi}^+; \forall b, \forall i$$

$$R_{bi} \geq 0 : \varepsilon_{bi}^-; \forall b, \forall i$$

# Model ...cont

- Proportional Reserve Constraint

$$-R_i \geq -x_i P_{gi} : \theta_i; \forall i$$

- Risk Generator Reserve Constraints

$$R^g \geq P_{gu} + R_u; \quad \forall u, u \in \text{Risk units}, \quad : \rho_u^g$$

$$\sum_i R_i \geq R^g : \rho$$

$R^g$  = Total reserve required to cover generator risk contingencies

# Duality with Example (Max Problem) Baumol/Wolf

## Primal Problem

$$\text{Max } Z_P = 6Q_1 + 2Q_2$$

$$\text{S.T. : } \begin{bmatrix} 4,1 \\ 3,2 \\ 1,1 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \leq \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}; Q_1, Q_2 \geq 0$$

## Dual Problem

$$\text{Min } Z_D = 5w_1 + 7w_2 + 3w_3$$

$$\text{S.T. : } \begin{bmatrix} 4,3,1 \\ 1,2,1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \geq \begin{bmatrix} 6 \\ 2 \end{bmatrix}; w_1, w_2 \geq 0$$

# Duality... contd

- The Dual objective function represents the total value of all the inputs which the firm has under its control.
- Dual constraints states that the value of the inputs going into the production of one unit of commodity in  $Q$  must be greater than or equal to the profit which the firm makes by producing that commodity.
- Must assign to each of the input in a commodity (output 1, say), a value sufficiently great to impute to them all the profits of output 1. Values Can't go infinitely high because it needs to minimise the Dual objective function.
- If both solutions are optimal,  $Z_P = Z_D$ . Converse is also true, if  $Z_P = Z_D$ , then both solutions are optimal.
- If  $Z_{P^*}$  and  $Z_{D^*}$  are respectively Primal and Dual feasible solutions, then  $Z_{P^*}$  will never exceed  $Z_{D^*}$ .

# Duality... contd

## Complementary Slackness

For a pair of optimal solutions of Primal and Dual,

- For  $i = 1, 2, \dots, m$ , the product of  $i$ th slack variable for P and the  $i$ th dual variable is zero.  $Q_{n+i} w_i = 0$ .  $Q_{n+i} = i$ th slack variable for the Primal problem
- Similarly for  $j = 1, 2, \dots, n$ , the product of  $j$ th slack variable for the dual problem and the  $j$ th variable for the primal problem is zero.

That means

- If  $i$ th slack variable of Primal is not zero, then  $i$ th dual variable must be zero
- Likewise, If  $j$ th slack variable of Dual is not zero, then  $j$ th Primal variable must be zero. (or both variables could be zero).

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# Duality ... contd

The value of  $Z_P = \sum_j c_j Q_j$  Will be maximised s.t inequality constraints  $\sum_j a_{ij} Q_j \leq b_i$  if the unconstrained Lagrangian expression  $L_\lambda = \sum_j c_j Q_j - \sum_i \lambda_i (\sum_j a_{ij} Q_j - b_i)$  also attains its maximum, and where the  $i^{\text{th}}$  Lagrangian multiplier  $\lambda_i = w_i$  is the optimal value of the  $i^{\text{th}}$  dual variable, so that the above Lagrangian becomes,

$$L_\lambda = \sum_j c_j Q_j - \sum_i \sum_j w_i a_{ij} Q_j + \sum_i b_i w_i$$

This theorem tells us that Lagrangian method of diff calculus can also be applied to LPP despite the fact that the constraints are inequalities, and changed into equations by using slack variables.

Marginal Profit  
contribution of  
input  $i$

$$\lambda_i = \frac{\partial Z_P}{\partial b_i}$$

# How to form Dual Price Equation: Wolf's Dual

Assign a (shadow) price (Lagrange multiplier /dual variable) to each constraint

Form a “Lagrangian” ( $L$ ) in which each constraint is multiplied by its price

Collect terms involving each primal variable to form a price equation corresponding to that variable

Re-arrange and substitute to form compact expressions for those “commodities” which are to be priced in the market

# Dual Formulation: Lagrangian Function

$$\begin{aligned}
 L = & \sum_{i \in n, b} (c_{bip} P_{gbi} + c_{bir} R_{bi}) + \sum_{(j \in N_i, j > i)} \tau_{ij} (B_{ij} (a_i - a_j) - P_{ij}) \\
 & + \sum_{(j \in N_i, j > i)} \psi_{ij} (-P_{ij} - P_{ji}) + \sum_i \lambda_i (\sum_{j \in N_i} P_{ij} - P_{gi} + P_{di}) + \\
 & + \sum_i \beta_i (P_{di}^{set} - P_{di}) + \sum_{(j \in N_i, j > i)} \phi_{ij}^+ (P_{ij} - P_{ij}^{\max}) \\
 & + \sum_{(j \in N_i, j > i)} \phi_{ij}^- (P_{ij}^{\min} - P_{ij}) + \sum_b \gamma_{bi}^+ (P_{gbi} - P_{gbi}^{\max}) - \sum_b \gamma_{bi}^- P_{gbi} \\
 & + \sum_i \sigma_i (P_{gi} - \sum_b P_{gbi}) + \sum_i \mu_i (R_i - \sum_b R_{bi}) \\
 & + \sum_b \varepsilon_{bi}^+ (R_{bi} - R_{bi}^{\max}) - \sum_b \varepsilon_{bi}^- R_{bi} + \sum_i \nu_i^+ (P_{gi} + R_i - P_{gi}^{cap-h}) \\
 & + \sum_i \nu_i^- (P_{gi}^{cap-l} - P_{gi} - R_i) + \sum_i \theta_i (R_i - x_i P_{gi}) \\
 & + \sum_u \rho_u^g (P_{gu} + R_u - R^g) + \sum_i \rho (R^g - \sum_i R_i) - \pi_s a_s
 \end{aligned}$$

# Forming Dual Price Equations

$$c_{bip} - \sigma_i + \gamma_{bi} = 0 \quad :P_{gbi}, \forall b$$

$$-\lambda_i + \sigma_i + \nu_i - x_i \theta_i + \rho_i^g * \delta_{pi} = 0 \quad :P_{gi}, \forall i$$

$$\delta_{pi} = 1 \text{ if } i \in u^b; u^b \in U; \text{ otherwise } = 0$$

$$c_{bir} - \mu_i + \varepsilon_{bi} = 0 \quad :R_{bi}; \forall b$$

$$\rho_i^g * \delta_{ri} - \rho + \mu_i + \nu_i + \theta_i = 0 \quad :R_i$$

$$\delta_{ri} = 1 \text{ if } i \in u^b \text{ and } u^b \in U; \text{ otherwise } = 0$$

$$-\sum_u \rho_u^g + \rho = 0 \quad :R^g; \forall i$$

$$\lambda_i - \beta_i = 0 \quad :P_{di}; \forall i$$

$$-\tau_{ij} + \lambda_i + \phi_{ij} - \psi_{ji} = 0 : P_{ij}; (\forall i, j \in N_i, j > i)$$

$$\text{where } \phi_{ij} = \phi_{ij}^+ - \phi_{ij}^-$$

$$\gamma_{bi} = \gamma_{bi}^+ - \gamma_{bi}^-; \quad \varepsilon_{bi} = \varepsilon_{bi}^+ - \varepsilon_{bi}^-; \nu_i = \nu_i^+ - \nu_i^-$$

$$\lambda_j - \psi_{ji} = 0 : P_{ji}$$

$$\sum_{j \in N_i, j > i} \tau_{ij} B_{ij} - \sum_{j \in N_i, j < i} \tau_{ji} B_{ji} = 0 : a_i, i \neq s$$

$$\pi_s = 0 : a_s, i = s$$

# Generation Prices

Collect terms to form price equations,

- Generation Price,

$$\lambda_i = c_{bip} + \gamma_{bi} + \nu_i - x_i \theta_i + \rho_i^g \cdot \delta_{pi}$$

$$\delta_{pi} = 1, \text{ if } i \in U^b; \text{ otherwise } = 0$$

- If  $i \in u$  i.e.,  $i$  generator risk reserve constraint is binding, and with no other constraints binding

$$\lambda_i = c_{bip} + \rho_i^g$$

- Net payment to generator  $i$  for its generation (energy) =  $\lambda_i - \rho_i^g$
- i.e., it gets  $\lambda_i$  for generation and pays  $\rho_i^g$  for the cost of its own reserve

# Reserve Prices

- **Reserve Price**

$$-\rho_i^g \cdot \delta_{ri} + \rho^g = c_{bir} + \varepsilon_{bi} + \nu_i + \theta_i$$

$$\rho^g = \sum_u \rho_u^g$$

The reserve price for generator  $i$  corresponds to:

1. The offer cost of a reserve block,  $c_{bir}$
2. The opportunity cost due to that reserve block's size limits,  $\varepsilon_{bi}$ , if binding.
3. The cost due to the joint capacity constraint for each individual unit  $\nu_i$ , if binding.
4. The cost due to the proportional reserve constraint for each individual unit  $\theta_i$ , if binding

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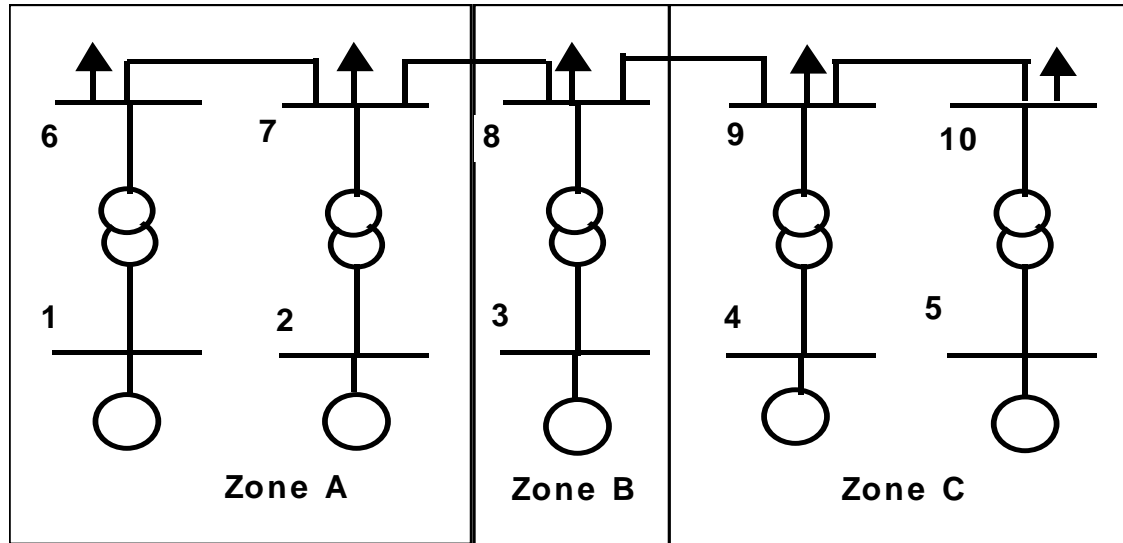
# Demand Prices

- Demand Price at Bus  $\beta_i = \lambda_i$
- Transmission congestion Effect  $\lambda_j = \lambda_i + \phi_{ij} - \tau_{ij}$

# Numerical Examples

- A 10 bus system is considered as shown in Figure. Buses 1 to 5 are generator buses and 6 to 10 are demand buses. Each line and each transformer has very high rating. There are 3 zones. Buses 1, 2, 6 and 7 are in zone A, buses 3 and 8 are in zone B, and buses 4, 5, 9 and 10 are in zone C. The demands are shown in the Table 1. The generation and reserve offers and their characteristics are shown in Table 2.
- Note that No Proportionality constraint is used in this example.
- Power Flow in circuit due to reserve is not included in the power flow equation

# Example Network



# Example contd ...

Bus	Demand, MW
6	100
7	50
8	100
9	150
10	200

Load Data. Table 1

Bus	Cpi	Cri	Pi (max)	Pi (cap)	Ri (max)
1	20	15	300	300	100
2	25	15	300	300	100
3	30	15	300	300	100
4	35	15	300	300	100
5	40	15	300	300	100

Generator Data: Table 2

# Results: No Transmission constraint

## Optimal Generation and Reserve Dispatch and Prices

	Energy , MW	Reserv e, MW	E-Rate, \$/MW	R-Rate, \$/MW	Gens are - Paid, Total \$
1	233.33		20	15	4666.6
2	233.33		25	15	5833.25
3	133.33	100	30	15	5499.9
4		100		15	1500
5		33.33		15	499.95
				Total	17999.7

Load Pays:  $(600) * 30 = \$18,000$

# Case 2: Transmission Constraint in line 8-9: Payments to generators

	Energy, MW	Reserve, MW	E-Rate, \$/MW	R-Rate, \$/MW	Gens are Paid, Total \$
1	174.995		20	15	3499.9
2	75.015	74.995	25	15	3000.3
3	0	100	25	15	1500
4	174.995		35	15	6124.825
5	174.995		40	15	6999.8
					21124.83

# Case 2: Transmission Constraint in line 8-9: Payments by Loads

	Demand, MW	Demand Price\$/M W	Loads Pay Total\$
6	100	25	2500
7	50	25	1250
8	100	25	2500
9	150	42.5	6375
10	200	42.5	8500
			21125

Congestion multiplier in line 8-9 = \$17.5 /MW.

Multiplier value = Price at bus 9 - Price at bus 8 = 42.5-25 = \$17.5 (verified)

# Effect of Transmission Constraint

- Case 2 has a higher generation and reserve cost of \$3125 due to network congestion.
- As a result of congestion, out of merit (uneconomic) generation has been dispatched, and
- Market has been separated with two sets of prices, \$42.5 / MW of energy at nodes 4, 5, 9,10 and \$25 at the all other nodes.

# End Of Presentation : Part 1

## Questions?

# Multiple SM Constraint Modelling at the Load Buses

- We define the Security Margin (SM) as a load margin in terms of MW from the current operating point. Margins are maintained by reserve.
- SM at each bus  $i$  during contingency,  $c$  is,

$$k_{i,c} = \frac{P_{di,c} - P_{di}}{P_{di,c}}; \forall c$$

$$P_{di} = 100 \text{ MW}$$

$$k_{i,c} = 0.2 (= 20\%)$$

For example,

$$P_{di,c} = \frac{P_{di}}{1 - k_{i,c}} = \frac{100}{0.8} = 125 \text{ MW}$$

# Multiple SM Constraint Modelling at the Wind Generator Buses

- Fluctuations in wind generations are modelled as security margins (SMs) at the wind generator buses. The margin in the model is maintained using reserve.
- SM at the Wind Generator Buses during contingency  $c$  is:

$$k_{gi,c} = \frac{P_{gi} - P_{gi,c}}{P_{gi}}; \forall c$$

For example:

$$P_{gi} = 100 \text{ MW}$$

$$k_{gi,c} = 0.3 \text{ (30\%)}$$

$$P_{gi,c} = 100 (1 - 0.3) = 70 \text{ MW}$$

# Multiple Security Margin Constrained Model (part)

- Objective function: *Min*
  - (minimise cost of generation and reserve)

$$Z = \sum_{i,b} (C_{b,i,p} P_{gbi} + C_{b,i,r} R_{b,i})$$

- *Subject to:*

$$P_{gi} - P_{di} = \sum_{j \in N_i} P_{ij} : \lambda_i; \forall i$$

- Energy balance
  - (normal and contingency states)

$$P_{gi,c} + R_{i,c} - P_{di,c} = \sum_{j \in N_i} P_{ij,c} : \lambda_{i,c}; i \in WG; \forall c$$

- Demand  
(normal and  
contingency states)

$$P_{di} = P_{di}^{set} : \beta_i; \forall i$$

$$P_{di,c} = \frac{P_{di}}{1 - k_{i,c}} : \beta_{i,c}; \forall i; \forall c$$

- Generation at the wind  
generator buses  
(normal and  
contingency states  
relationship)  
Etc....

$$P_{gi,c} = P_{gi} (1 - k_{gi,c}) : \xi_{gi,c}; \forall i, \forall c$$

# Forming Dual Price Equations

Form Lagrangian:

$$\begin{aligned}
 L = & \sum_{i \in n, b} (c_{bip} P_{gbi} + c_{bir} R_{bi}) + \dots + \sum_i \lambda_i (\sum_j P_{ij} - P_{gi} + P_{di}) \\
 & + \sum_{i, c} \lambda_{ic} (\sum P_{ij, c} - P_{gi, c} + P_{di, c}) + \sum_{i, c} \zeta_{gi, c} (P_{gi} (1 - k_{gi, c}) - P_{gi, c}) \\
 & + \sum_i \beta_i (P_{di}^{set} - P_{di}) + \sum_{i, c} \beta_{ic} \left( \frac{P_{di}}{1 - k_{i, c}} - P_{di, c} \right) + \dots
 \end{aligned}$$

Collect terms to form price equations:

$$\lambda_i - \beta_i + \sum_c \frac{\beta_{ic}}{1 - k_{ic}} = 0 : P_{di}; \forall i$$

$$\lambda_{ic} - \beta_{ic} = 0 : P_{di, c}; \forall i$$

*etc....*

Re-arrange....

# Generation Price Signal at Node $i$

$$\lambda_i + \sum_c \lambda_{i,c} - \sum_c \lambda_{i,c} \cdot k_{gi,c} - \rho_i^g \cdot \delta_{pi} = c_{bip} + \gamma_{bi} + \nu_i - x_i \cdot \theta_i$$

Generation at  $i$  receives:

- **The shadow price on its nodal power balance constraint during normal conditions**
- **(Plus) the combined shadow prices valuing its contribution to meeting load during the different contingencies**
- **(Minus) cost of reserve to maintain margins at the Wind Generation bus (i.e., Generator price is reduced)**
- **(Minus) cost of reserve to cover the risk of its own generation (i.e., Generators Pay)**

# Demand Price Signal at Bus $i$

$$\lambda_i + \sum_c \frac{\lambda_{ic}}{1 - k_{ic}}$$

Load at  $i$  pays:

- The shadow price on its nodal power balance constraint during normal conditions
- (Plus) the combined shadow prices measuring the cost of meeting load during the different contingencies...

*... weighted by the relevant 'load variation factor'*

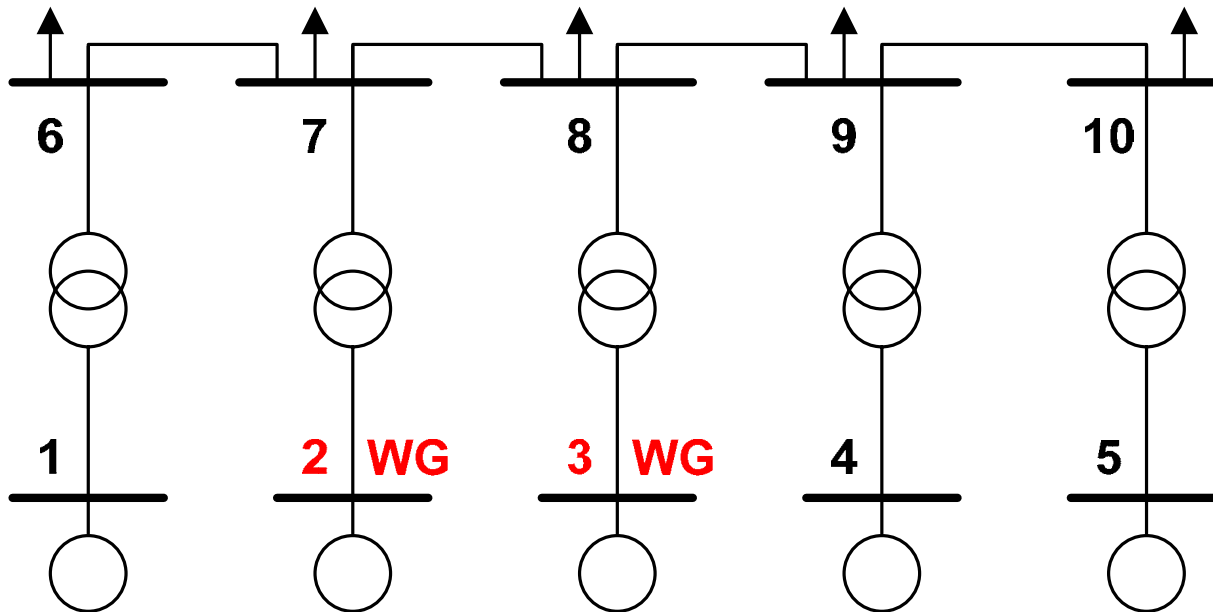
# Reserve Price Signal at Bus $i$

$$\sum_c \lambda_{i,c} + \sum_{u \in U^b} \rho_u^g - \rho_i \delta_{ri} = c_{bir} + \varepsilon_{bi} + \nu_i + \theta_i$$

Reserve at  $i$  receives:

- The reserve price at bus  $i$  is given by the multiplier associated with the constraints defining reserve dispatch to meet the Security Margins;
- (Plus) multipliers associated with the constraints defining generator risk reserve,
- (Minus) the cost of reserve to cover the risk generator's own generation

# Numerical Example



Case 1: Only multiple security margins at the load buses are considered.

Case 2: Multiple security margins both at the load buses and the wind generator buses are considered.

Load Bus	C1	C2	C3	Load MW
6	0.3	0.3	0.3	125
7	0.3	0.2	0	50
8	0.2	0.2	0.3	100
9	0.15	0.2	0.2	150
10	0.2	0.2	0.3	200

**SM for the contingencies at the Load Buses and Loads**

Gen Bus	Pg, \$/MW	Rg, \$/MW	Pg-max, MW	cap, MW	Rg_max, MW
1	35	15	300	300	200
2	20	15	100	100	0
3	25	15	100	100	0
4	30	15	300	300	200
5	40	15	300	300	200

**Generator Offers and Data**

Gen Bus	C1	C2	C3
1	0	0	0
2	0.2	0.2	0.2
3	0.1	0.1	0.1
4	0	0	0
5	0	0	0

**SM for the contingencies at the Generator Buses**

Bus	Load, MW	Dem Price \$/MW	Load pays, \$
6	125	41.43	5178.57
7	50	35.00	1750.00
8	100	38.75	3875.00
9	150	38.75	5812.50
10	200	41.43	8285.71
Total	625		24901.79

Demand Prices and Payments  
by Loads: Case 1

Bus	PG, MW	Reserve, MW	Gen price, \$/MW	Reserve Price, \$/MW	Payment to Gen, \$
1	125	175	35	15	7000
2	100		35	15	3500
3	100		35	15	3500
4	300		35	15	10500
5	0	26.786	35	15	401.79
Total					24901.79

Optimal Dispatch and Payments  
to Generators: Case 1

The  $\lambda_i$ 's during the normal condition for all buses are \$20.  $\forall i$

The contingency C3 is binding for all buses with  $\lambda_{i,c3} = \$15$

The generation price for all generators =  $\lambda_i + \lambda_{i,c3} = \$20 + \$15 = \$35$

Bus	PG, MW	Reserve MW	Gen price, \$/MW	Res Price, \$/MW	Payment to Gen
1	125	175	35	15	7000
2	100		32	15	3200
3	100		33.5	15	3350
4	300		35	15	10500
5	0	56.786	35	15	851.79
Total					24901.79

Optimal Dispatch and Payments to Generators:  
Case 2

The generation price for the wind generators is  $\lambda_i + \sum_c \lambda_{i,c} - \sum_c \lambda_{i,c} \cdot k_{gi,c}$

Demand prices in this case are same as in case 1

$$\lambda_i = \$20, \lambda_{i,c3} = \$15$$

# Summary

- Load price is not the same as for generation, because:
  - Load places different costs on the system depending on how much its load variation contributes to each contingency
  - Load prices increase to maintain these security margins
  - The cost of reserves is realised from loads
- Those costs occur as a result of having to:
  - Adjust generation dispatch under normal conditions and/or
  - Provide reserve response for contingency conditions
- The wind generation nodal prices get reduced while maintaining the security margins at these buses using reserve to cover generation fluctuation
- The cost of reserve is realised from wind generators through reduction of generation price there

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**End of Presentation**