Lecture 3

Dispatch and Pricing Model with Network

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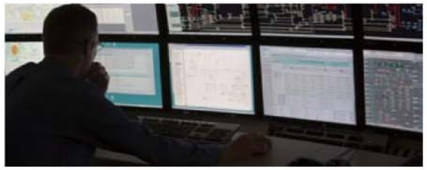
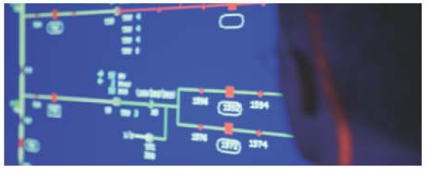
TRANSPOWER



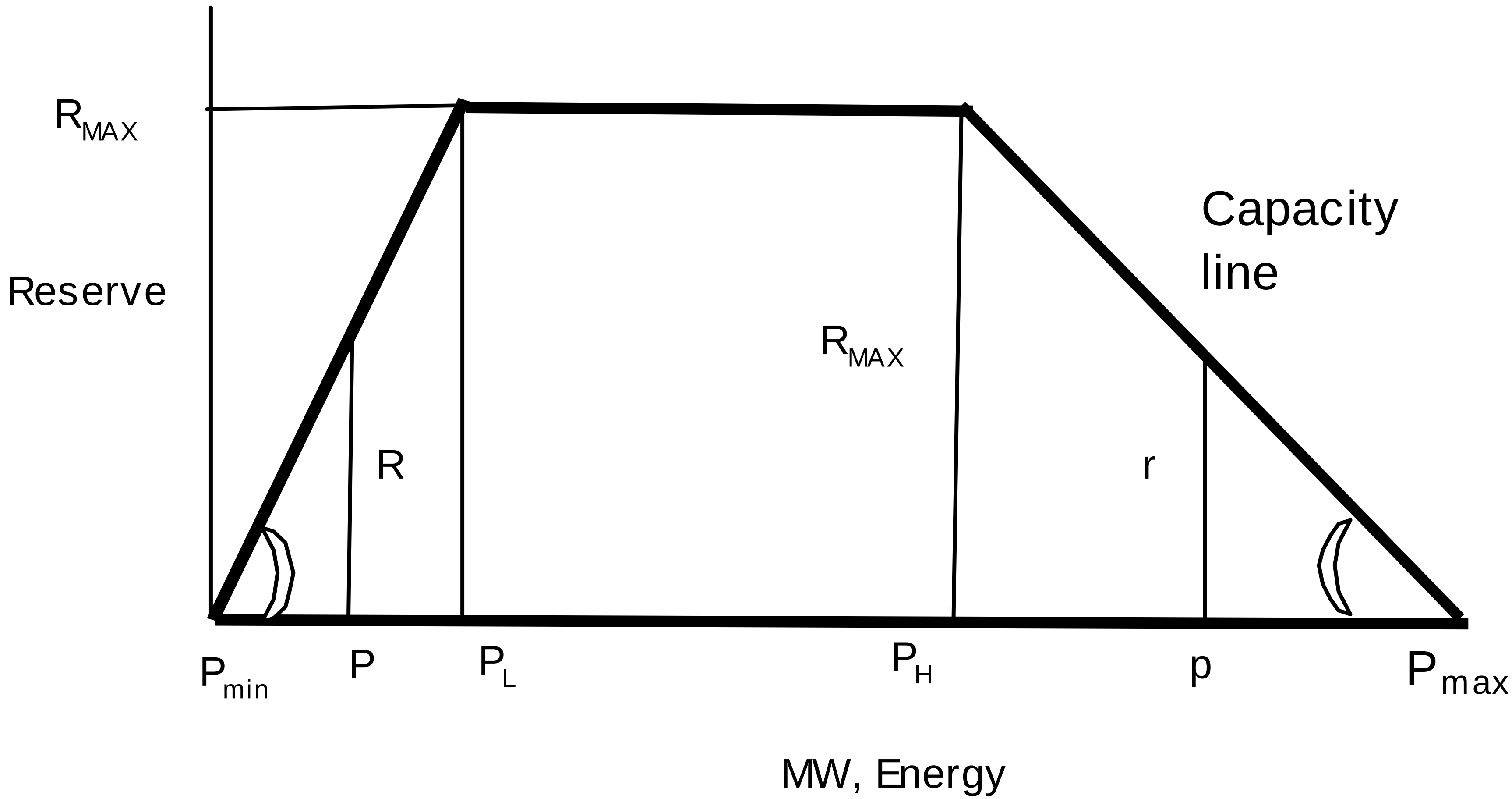


SYSTEM OPERATOR

Issues Addressed in the Pricing Problem

- Representation of Generation-Risk constraint in a dispatch model,
- Representation of Loss less Network
- Develop Primal model and its decomposition by dual analysis, and ;
- Effects of these constraints on the Generation, Reserve and the Demand prices.

Reserve Model



Reserve constraints

- Proportional Constraint $R_i \leq b_i . P_i$
- Reserve upper bound Constraint $R_i \leq R_i^{\max}$
- Generator Joint capacity Constraint $P_i + R_i \leq P_i^{cap}$
- Generator upper and lower bound Constraint $P_i \geq 0$, $P_i \leq P_i^{\max}$

Generator Risk Modelling

- Cover the loss of any of the largest ‘Risk units’
- Size of the risk be optimised,
- Any reserve on an unit can not be counted when considering the reserve needed to cover that particular unit,

$$\sum_i R_i \geq P_u + R_u \;;\; \forall u,\; u \in \textit{Risk units}$$

P_u = Cleared Generation of Risk generator, u

R_u = Cleared Reserve of Risk generator, u

$\sum_i R_i$ = Total requirement of reserve from all generators

Primal DC Model

- Objective Function min

generation cost reserve cost

$$Z = \sum_{i,b} (C_{b,i,p} P_{gbi} + C_{b,i,r} R_{b,i})$$

$C_{b,i,p}$ = Energy offer price of block b by generator i \$/MW

P_{gbi} = Quanity offered for energy of block b by generator i,MW

$C_{b,i,r}$ = Reserve offer price of block b by generator i \$/MW

$R_{b,i}$ = Quanity offered for reserve of block b by generator i,MW

Model … contd

- We define a set N_i ={j: there is a line between i and j}. Power flows in the line between i and j during normal condition are given by:

$$P_{ij} = B_{ij}(a_i - a_j) : \tau_{ij}; (\forall i, j \in Ni; j > i)$$

$$P_{ji} = -P_{ij} : \psi_{ij}; (\forall i, j \in Ni; j > i)$$

- Bij = Admittance in pu
- ai = Bus angle in radian

Model contd...

Energy Balance

$$P_{gi} - P_{di} = \sum_{j \in Ni} P_{ij} : \lambda_i ; \forall i$$

P_{di} = demand at the load bus i

λ_i = Dual variable associated with the power balance equation at bus i

j€Ni All nodes connected to bus i, as defined earlier

Note that network Loss is not considered in the model

Swing Bus Angle

$$a_i = 0 : \pi_i ; i = s$$

Model continued

- Demand $$P_{di} = P_{di}^{set} : \beta_i ; \forall i$$

▲ Line Flow Limits

$$-P_{ij} \geq -P_{ij}^{\max} : \phi_{ij}^{+} ; (\forall i, j \in N_i ; j > i)$$

$$P_{ij} \geq P_{ij}^{min} : \phi_{ij}^{-} ; (\forall i, j \in N_i ; j > i)$$

Model continued ...

- Cleared Generation Offer blocks and Gen Dispatch

$$-P_{bi} \geq -P_{bi}^{\max} : \gamma_{bi}^{+}; \forall b, \forall i$$

$$\sum_{b} P_{bi} - P_{gi} = 0 : \sigma_{i}; \forall i$$

$$P_{bi} \geq 0 : \gamma_{bi}^{-}; \forall b, \forall i$$

- Joint generation and reserve $-P_i - R_i \geq -P_i^{cap} \quad : \upsilon_i, \forall i$
- Cleared Reserve Offer blocks and Reserve Dispatch

$$\sum_{b} R_{bi} - R_{i} = 0 : \mu_{i}; \forall i$$

$$-R_{bi} \geq -R_{bi}^{\max} : \varepsilon_{bi}^{+}; \forall b, \forall i$$

$$R_{bi} \geq 0 : \varepsilon_{bi}^{-}; \forall b, \forall i$$

Model continued ...

- Proportional Reserve Constraint

$$-R_i \geq -x_i P_{gi} : \theta_i ; \forall i$$

- Risk Generator Reserve Constraints

$$\sum_i R_i \geq R^g : \rho$$

$$R^g \geq P_{gu} + R_u ; \quad \forall u, \ u \in Risk \ units, \quad : \rho_u^g$$

R^g = Total reserve required to cover generator risk contingencies

Duality and an example ..

- Duality refers to the fact that every linear problem has associated with it another linear programming problems.

- New variables in dual problem called dual variables, multiplier or shadow prices

- <u>Optimal</u> values of Primal and Dual Objective functions are equal.

How to form Dual Price Equation: Wolf's Dual

Assign a (shadow) price (Lagrange multiplier /dual variable) to each constraint

Form a "Lagrangian" *(L)* in which each constraint is multiplied by its price

Collect terms involving each primal variable to form a price equation corresponding to that variable

Re-arrange and substitute to form compact expressions for those "commodities" which are to be priced in the market

Dual

$$c_{bip} - \sigma_i + \gamma_{bi} = 0 \quad :\mathrm{P}_{gbi}, \forall b$$

$$-\lambda_i + \sigma_i + \upsilon_i - x_i\theta_i + \rho_i^g * \delta_{pi} = 0 \quad :P_{gi}, \forall i$$

$$\delta_{pi} = 1 \text{ if } i \in u^b;\ \mathrm{u}^{\mathrm{b}} \in \mathrm{U};\ \text{otherwise } = 0$$

$$c_{bir} - \mu_i + \varepsilon_{bi} = 0 \quad :R_{bi}; \forall b$$

$$\rho_i^g * \delta_{ri} - \rho + \mu_i + \upsilon_i + \theta_i = 0 \quad :R_i$$

$$\delta_{ri} = 1 \text{ if } i \in u^b \text{ and } \mathrm{u}^{\mathrm{b}} \in \mathrm{U};\ \text{otherwise } = 0$$

$$-\sum_u \rho_u^g + \rho = 0 \quad :R^g; \forall i$$

$$\lambda_i - \beta_i = 0 \quad :P_{di}; \forall i$$

$$-\tau_{ij} + \lambda_i + \phi_{ij} - \psi_{ji} = 0 : P_{ij}; (\forall i, j \in N_i, j > i)$$

$$\text{where } \phi_{ij} = \phi_{ij}^+ - \phi_{ij}^-$$

$$\gamma_{bi} = \gamma_{bi}^+ - \gamma_{bi}^-; \quad \varepsilon_{bi} = \varepsilon_{bi}^+ - \varepsilon_{bi}^-; \upsilon_i = \upsilon_i^+ - \upsilon_i^-$$

$$\lambda_j - \psi_{ji} = 0 : P_{ji}$$

$$\sum_{j \in N_i, j > i} \tau_{ij} B_{ij} - \sum_{j \in N_i, j < i} \tau_{ji} B_{ji} = 0 : a_i, i \neq s$$

$$\pi_s = 0 : a_s, i = s$$

Generation Prices

- Generation Price,

$$\lambda_i = c_{bip} + \gamma_{bi} + \upsilon_i - x_i \theta_i + \rho_i^g . \delta_{pi}$$

If $i \in u$ i.e., i *generator risk reserve constraint is binding, and with no other constraints binding* $\delta_{pi} = 1, \text{ if } i \in U^b; \quad \text{otherwise} \quad = 0$

$$\lambda_i = c_{bip} + \rho_i^g$$

Net payment to generator *i* for its generation (energy) = $\lambda_i - \rho_i^g$

i.e., it gets λ_i for generation and pays ρ_i^g for the cost of its own reserve

Reserve Prices

Reserve Price

$$-\rho_i^g . \delta_{ri} + \rho^g = c_{bir} + \varepsilon_{bi} + \upsilon_i + \theta_i$$

$$\rho^g = \sum_u \rho_u^g$$

The reserve price for generator *i* corresponds to:

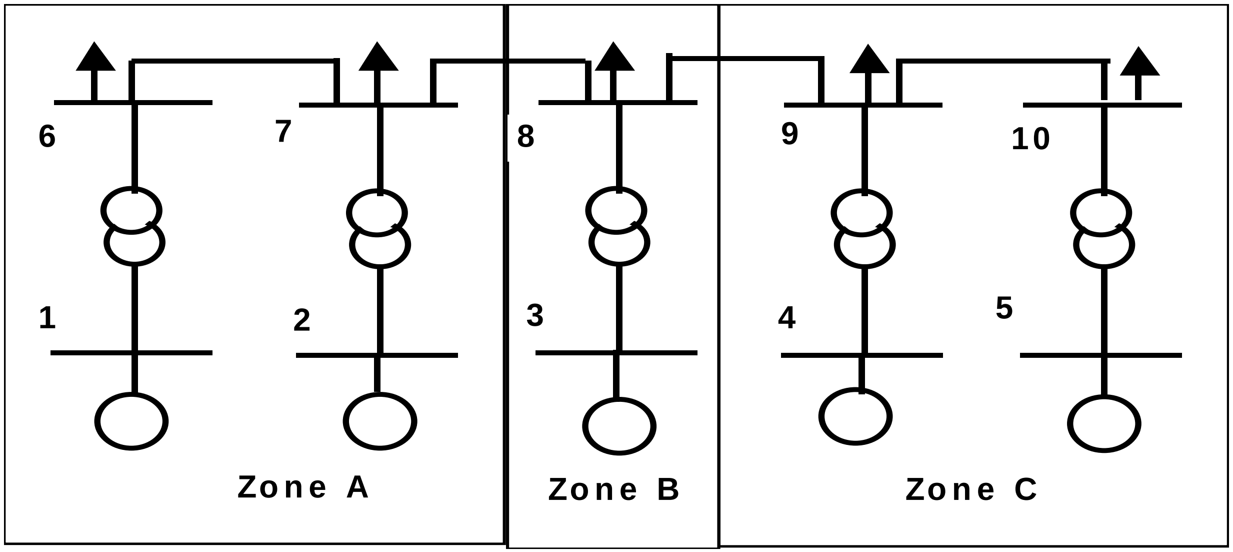
1. The offer cost of a reserve block, c_{bir}
2. The opportunity cost due to that reserve block's size limits, ε_{bi}, if binding.
3. The cost due to the joint capacity constraint for each individual unit υ_i ,if binding.
4. The cost due to the proportional reserve constraint for each individual unit θ_i ,if binding

Demand Prices

- Demand Price at Bus $\beta_i = \lambda_i$

NUMERICAL EXAMPLES

- A 10 bus system is considered as shown in Figure. Buses 1 to 5 are generator buses and 6 to 10 are demand buses. Each line and each transformer has very high rating. There are 3 zones. Buses 1, 2, 6 and 7 are in zone A, buses 3 and 8 are in zone B, and buses 4, 5, 9 and 10 are in zone C. The demands are shown in the Table 1. The generation and reserve offers and their characteristics are shown in Table 2.
- Note that No Proportionality constraint is used in this example.
- Power Flow in circuit due to reserve is not included in the power flow equation.


6
7
8
9
10
1
2
3
4
5
Zone A
Zone B
Zone C

NUMERICAL EXAMPLES

Load Data (ignore C1, C2 and C3 columns): Table 1

Bus	Dem	C1	C2	C3
6	100	30	30	30
7	50	30	20	0
8	100	20	20	20
9	150	15	20	20
10	200	20	20	30

Generator Data: Table 2

Bus	Cpi	Cri	Pi (max)	Pi (cap)	Ri (max)
1	20	15	300	300	100
2	25	15	300	300	100
3	30	15	300	300	100
4	35	15	300	300	100
5	40	15	300	300	100

Results: No Transmission constraint

Optimal Generation and Reserve Dispatch and Prices

	Energy, MW	Reserve, MW	E-Rate, $/MW	R-Rate, $/MW	Gens are -Paid, Total $
1	233.33		20	15	4666.6
2	233.33		25	15	5833.25
3	133.33	100	30	15	5499.9
4		100		15	1500
5		33.33		15	499.95
				Total	**17999.7**

Load Pays: (600)* 30 = $18,000

Case 2: Transmission Constraint in line 8-9: Payments to Generators

	Energy, MW	Reserve, MW	E-Rate, $/MW	R-Rate, $/MW	Gens are Paid, Total $
1	174.995		20	15	3499.9
2	75.015	74.995	25	15	3000.3
3	0	100	25	15	1500
4	174.995		35	15	6124.825
5	174.995		40	15	6999.8
					21124.83

Case 2: Transmission Constraint in line 8-9: Payments by Loads

	Demand, MW	Demand Price$/MW	Loads Pay Total$
6	100	25	2500
7	**50**	25	1250
8	100	25	2500
9	150	42.5	6375
10	200	42.5	8500
			21125

Congestion multiplier in line 8-9 = $17.5 /MW.

Multiplier value = Price at bus 9 - Price at bus 8 = 42.5-25 = $17.5 (verified)

Effect of Transmission Constraint

- Case 2 has a higher generation and reserve cost of $3125 due to network congestion.

- As a result of congestion, out of merit (uneconomic) generation has been dispatched, and

- Market has been separated with two sets of prices, $42.5 / MW of energy at nodes 4, 5, 9,10 and $25 at the all other nodes.

Example: Effect of transmission congestion

Two node network with no loss. See Figure A2.

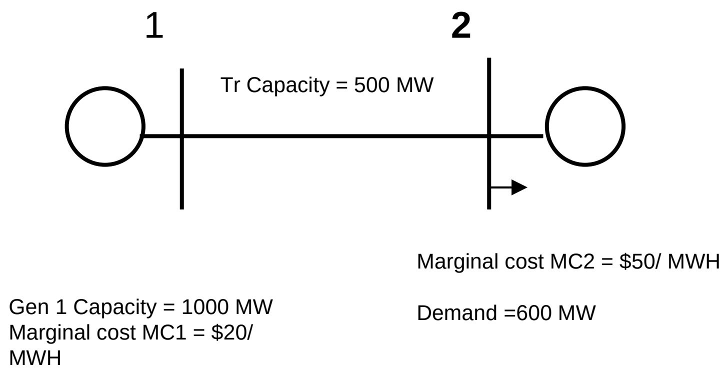


Figure A2: Two node loss less network

No transmission Congestion

- Spot price (system marginal price) at node 1 & 2 $\rho 1 = \rho 2 =$ $20 / MWH.
- Total generation cost = 600 *20= $12000
- Total bill paid by the customer = 600*20 = 12,000

With Transmission Congestion for a transmission capacity = 500 MW

- Since transmission line is congested, a marginal electricity demand at node 2 can be met only by using expensive power generated at node 2.
- Marginal price at node 2, $\rho 2$ = 50 /MWH.
- Total generation cost = 500 * 20 + 100*50 = $15,000.
- Total bill paid by the customer = 600*50 = 30,000
- **Cost of Congestion = 15000 – 12000 = $3000**
- Congestion Rent = 30,000 -15,000 = 15,000

Difference in Generation and Reserve cost due to congestion

[Case 2 -Case 1]

Gen	Case2-Case1(E)	Rate	Diff
1	-58.335	20	-1166.7
2	-158.315	25	-3957.88
3	-133.33	30	-3999.9
4	174.995	35	6124.825
5	174.995	40	6999.8
Energy			4000.15
Reserve			-874.95
NET diff			3125.2